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# MODIFICATIONS TO THE FINITE ELEMENT MODELLER PROGRAM "REFINE"

by  
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## CONTRACTOR REPORT

Prepared for

**Defence  
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Scientific Authority

  
Neil Pegg for D.R. Smith

August 1990

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## ABSTRACT

Program REFINE was initially developed as part of an adaptive finite element modelling package for VAST where from estimates of discretization errors based on element residuals, the finite element model would be selectively refined (by element subdivision) until acceptable levels of accuracy (according to computed error estimates) were attained [1]. However, increasingly the REFINE program finds application as a general modelling tool which complements the data generation capabilities of PFAST and HFAST. Under these circumstances, the PFAST and HFAST programs need only concentrate on achieving accurate geometric models, since subsequent refinement of the model to attain acceptable finite element accuracy can be achieved with REFINE. This is particularly useful when the initial model is defined in a pointwise fashion with a graphics table. In this context as well, selective model refinement would generally be employed but the experience of the modeller would dictate where the model refinements would be.

## RÉSUMÉ

Le logiciel REFINE a été conçu dans le cadre d'un progiciel adaptatif de modélisation aux éléments finis pour VAST. Le programme raffine sélectivement (par subdivision des éléments) le modèle aux éléments finis à partir d'estimations des erreurs de discrétisation basées sur les résidus des éléments, jusqu'à ce que le niveau de précision indiqué par les estimations d'erreur calculées soit atteint [1]. Toutefois, les applications du programme REFINE se multiplient en tant qu'outil général de données de PFAST et de HFAST. Dans ces circonstances, les programmes PFAST et HFAST peuvent n'être utilisés que pour produire des modèles géométriques précis, REFINE permettant de raffiner ultérieurement le modèle pour obtenir des éléments finis de précision acceptable. Cela est particulièrement utile lorsque le modèle initial est défini point par point à l'aide d'une tablette graphique. Dans ce contexte également, le modélisateur aurait recours au raffinement sélectif du modèle, mais son expérience lui dicterait où apporter les raffinements.

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## CHAPTER 1 INTRODUCTION

### 1.1 Introduction

Program REFINE was initially developed as part of an adaptive finite element modelling package for VAST where from estimates of discretization errors based on element residuals, the finite element model would be selectively refined (by element subdivision) until acceptable levels of accuracy (according to computed error estimates) was attained [1]. However, increasingly the REFINE program finds application as a general modelling tool which complements the data generation capabilities of PVASt and HVASt. Under these circumstances, the PVASt and HVASt programs need only concentrate on achieving accurate geometric models, since subsequent refinement of the model to attain acceptable finite element accuracy can be achieved with REFINE. This is particularly useful when the initial model is defined in a pointwise fashion with a graphics tablet. In this context as well, selective model refinement would generally be employed but the experience of the modeller would dictate where the model refinements would be.

This report describes recent work completed to extend the capabilities of the program REFINE. Chapter 2 describes the program modifications which enable the refinement of additional element types. Almost all element types are operational now. Chapter 3 describes the introduction of appropriate cubic multi-point constraint equations for refinement of stiffened or unstiffened panels. Chapter 4 summarizes deficiencies in refinement of substructured models and discusses how one of them was addressed by automatically defining newly generated boundary nodes (on superelement boundaries) as either slave nodes or master nodes in a manner that preserves maximum structural flexibility. Chapter 5 describes a restart capability for REFINE suitable for either substructured or unstructured finite element models. Chapter 6 outlines other

improvements in the superelement refinement algorithm. Chapter 7 describes several miscellaneous changes to the REFINE program not called for in the contract statement of work but which were determined to be desirable in recent applications of the program.

## CHAPTER 2 REFINEMENT OF ADDITIONAL ELEMENT TYPES

### 2.1 Introduction

The element refinement algorithm of the program REFINE has been extended to several new elements from the VAST element library including the curved beam element (IEC=7), the axisymmetric shell (IEC=19) and the second degenerate form of the brick, namely the square based pyramid (IEC=2). In general, element refinement involves the generation of new nodes, the removal of the connectivity of the parent element to be refined, the generation of connectivities for new elements produced by the refinement process and also the generation and manipulation of constraint equations for irregular nodes on the interface between refined elements and unrefined elements. The program modifications for each of the above mentioned element types will be discussed separately in the following sections.

### 2.2 Square Based Pyramid (IEC=2)

The square based pyramid is a special case of the solid element and is defined by appropriate repetition of node numbers in the element connectivity list (see Figure 2.1).

The generation of new nodes for the square based pyramid followed an approach similar to that taken for the refinement of the tetrahedron elements and posed no great difficulties. The generation and manipulation of the constraint equations also did not create any unusual problems. The generation of the connectivities produced by the element refinement however, is a complex procedure. The philosophy of the REFINE program has been that the refinement of an element normally results in the generation of only smaller elements of the same type. The one exception to this is the refinement of the transition element which results in

the creation of transition and shell elements as will be discussed in the following section. The refinement of the square based pyramid is also an exception to this rule since the refinement results in the formation of square based pyramids and tetrahedron elements. Therefore, a new element group is necessarily produced. The treatment of this new group is similar to the new group formed by refinement of the transition element. The refinement of the square based pyramid poses an additional problem since the total number of elements produced does not follow the general equation:

$$NELMF = NOR^{IDIEC} \quad (2.1)$$

where

NELMF = total number of elements produced by refinement

NOR = order of refinement

IDIEC = spatial dimension of the element type (1,2 or 3)

In this case, IDIEC is 3 since the square based pyramid is a three-dimensional element. The number of pyramid elements produced is given by the following equation:

$$NELMP = \frac{2}{3} * [NOR^{IDIEC} + \frac{NOR}{2}] \quad (2.2)$$

where

NELMP = total number of pyramid elements produced by refinement

NOR = order of refinement

IDIEC = 3

The number of tetrahedron elements produced is given by the following equation:

$$NELMT = \frac{2}{3} [NOR^{IDIEC} - NOR] \quad (2.3)$$

where

NELMT = total number of tetrahedron elements produced by refinement  
NOR = order of refinement  
IDIEC = 3

The refinement of the square based pyramid does produce an equivalent number of elements to Equation 2.1 if half the number of tetrahedrons given by Equation 2.3 are added to the total number of pyramids given by Equation 2.2. This manipulation is somewhat justified with the observation that each pyramid may be broken into 2 component tetrahedrons. Table 1 summarizes element information for several different orders of refinement. It can be readily seen that combining Equation 2.2 with half of Equation 2.3 always yields Equation 2.1.

Now that the number and type of elements produced have been established, the next consideration is the orientation of the elements. The orientation of elements is important when generating the load data for the refined model. The new pyramid elements are therefore generated with their face orientation the same as the parent element. The new tetrahedron elements produced on each of the faces have consistently one orientation. The logic required in the program to generate the element connectivities is quite complex and being sufficiently unique, it forms a separate section in subroutine REFIN. Load refinement for square based pyramids has not been performed under this contract.

### 2.3 Transition Element (IEC=6)

Refinement of the transition element presents some unique difficulties and some further development is still required. Presently, a transition element refinement of degree  $n$  generates  $n$  transition elements and  $(n-1) \times n$  shell elements. The total number of elements produced is governed by Equation 2.1. As illustrated in Figure 2.2 for a refinement degree of 2, the face adjoining a solid element is divided into  $n$  ele-

ments with no element divisions through the element thickness. Element refinements of the adjacent solid element with refinement degree  $n$ , on the otherhand, results in  $n$  element divisions through the thickness and thus a refinement inconsistency on the common interface. In fact, depending whether the transition or solid element is refined first, difficulties are sometimes experienced in generating correct multi-point constraint equations on the transition-to-solid element interface.

Although a coding error was previously suspected, as the cause of the difficulty in generating correct MPC equations, this has now been ruled out. After considerable study, it became apparent that for reliable MPC equations the MPC on the transition-to-solid element interface modification of the MPC equation generation algorithm for the solid element was unavoidable. Different MPC equations would be generated on the faces of a solid element depending on whether a transition element or another solid element attached to it. In order to achieve this requires considerable changes to the databases on which REFINES operates. Information related to the neighbours of each element in the model would have to be generated and manipulated during refinement of a model. It is not difficult to appreciate the difficulty of the task especially when one contemplates the refinement of a model which has been refined previously and includes MPC equations on the interface between refined and unrefined sections. The philosophy of the REFINES program has been that each element selected by the user can be refined and appropriate MPC equations generated without regard to its neighbours.

An alternative refinement strategy involving the division of a transition element into smaller transition elements and wedge elements as illustrated in Figure 2.3 has therefore been considered. Although this has the advantages that any given interface is always divided consistently and that the solid element zone moves slightly into the shell structure, some deterioration of element proportioning develops if extensive refinements in the original transition element are required. However, if

the maximum refinement order is no more than five or six and the maximum aspect ratio (element length to thickness) is no greater than ten, the aspect ratio will be acceptable.

The work performed under this contract consisted of a review of the existing coding to determine the most appropriate treatment of the transition element. It can be concluded that refinement of transition elements cannot be provided within the framework of REFINE without compromising the assumptions on which the program presently operates. The first refinement strategy involving changes to the program data structures represents a major undertaking and should not be adopted unless adequate manpower can be budgeted.

#### 2.4 Curved Beam Element (IEC=7) and Axisymmetric Shell (IEC=19)

The curved beam element (superparametric beam element) (IEC=7) is suitable as a stiffener for the thick/thin shell element (IEC=1) and if used as such, the displacement node numbers of the shell to which the beam is connected are to be identified. These nodes must be identified in the REFINE program since any constraint equations generated must relate to the displacement nodes and not to the geometric nodes. The same procedure for identifying the displacement nodes is also necessary for the axisymmetric shell element (IEC=19) since the constraint equations relate to those nodes. The refinement of the curved beam elements also required the development of a new subroutine CBEAM to calculate the curved beam properties at new nodes using the shape function of each new nodal point in question.



TABLE 2.1

Number of Elements Produced by  
Refinement of a Square Based Pyramid

NOR Order of Refinement	NELMF General IDIEC=3 (Equation 2.1)	NELMP Pyramid Elements (Equation 2.2)	NELMT Tetrahedron Elements (Equation 2.3)
1	1	1	0
2	8	6	4
3	27	19	16
4	64	44	40

## CUBE MODELLED BY SQUARE BASED PYRAMID ELEMENTS

```

0
27 27 1
1 10.0000 0.0000 10.0000
2 10.0000 0.0000 5.0000
3 10.0000 0.0000 0.0000
4 10.0000 5.0000 10.0000
5 10.0000 5.0000 5.0000
6 10.0000 5.0000 0.0000
7 10.0000 10.0000 10.0000
8 10.0000 10.0000 5.0000
9 10.0000 10.0000 0.0000
10 5.0000 0.0000 10.0000
11 5.0000 0.0000 5.0000
12 5.0000 0.0000 0.0000
13 5.0000 5.0000 10.0000
14 5.0000 5.0000 5.0000
15 5.0000 5.0000 0.0000
16 5.0000 10.0000 10.0000
17 5.0000 10.0000 5.0000
18 5.0000 10.0000 0.0000
19 0.0000 0.0000 10.0000
20 0.0000 0.0000 5.0000
21 0.0000 0.0000 0.0000
22 0.0000 5.0000 10.0000
23 0.0000 5.0000 5.0000
24 0.0000 5.0000 0.0000
25 0.0000 10.0000 10.0000
26 0.0000 10.0000 5.0000
27 0.0000 10.0000 0.0000

```

```

2 3
0.300E+08 0.300E+00 0.737E-03

```

```

9 19 19 3 14 19 11 6 7 19 19 1 13 19 10 4 8 19 19 2
27 19 19 21 23 19 20 24 9 19 19 3 14 19 11 6 18 19 19 12
25 19 19 27 22 19 23 26 7 19 19 9 13 19 14 8 16 19 19 18

```

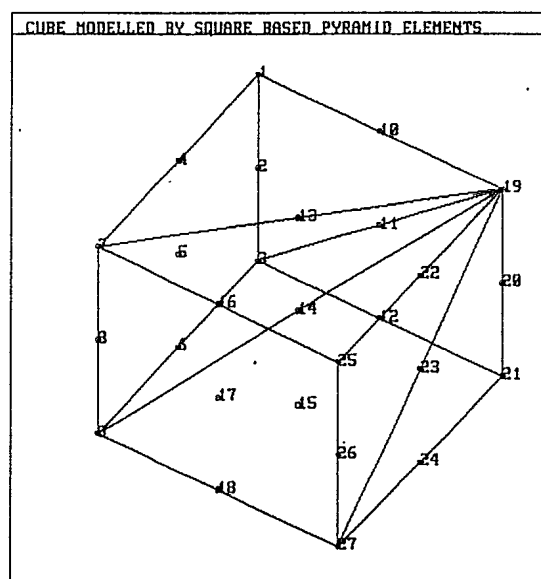


FIGURE 2.1: Typical GOM File for VAST With Square-Based Pyramid Elements.

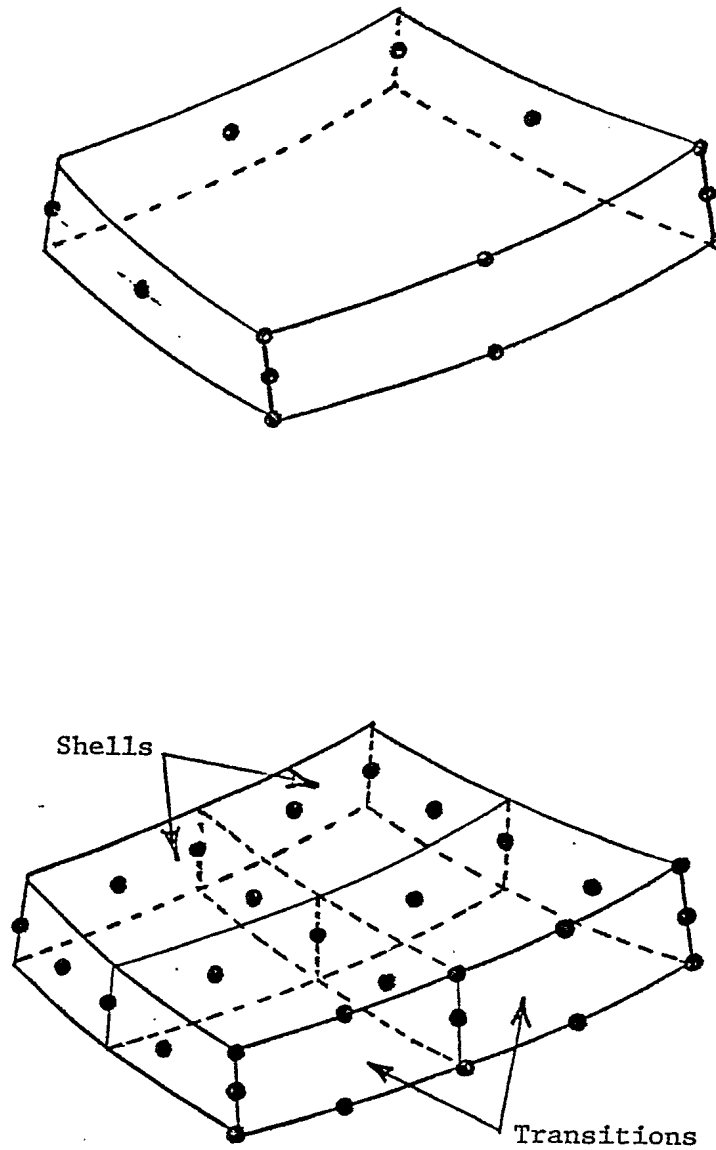


FIGURE 2.2: Transition Element Refinement Strategy as Originally Implemented in REFINE.

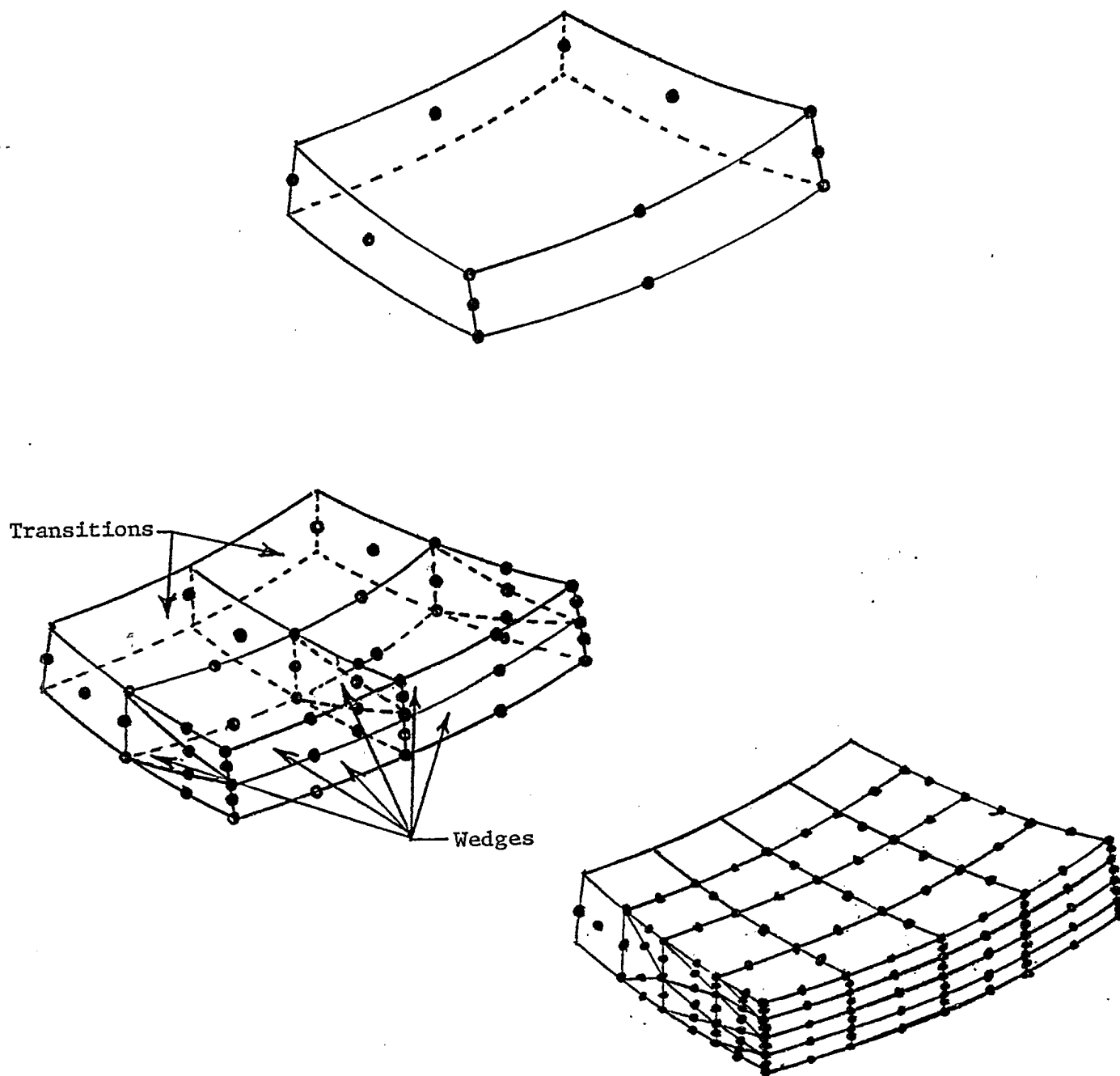


FIGURE 2.3: Alternate Transition Element Refinement Strategy  
Producing Wedge Elements.



CHAPTER 3  
CUBIC MULTIPOINT CONSTRAINT EQUATIONS FOR REFINEMENT  
OF STIFFENED OR UNSTIFFENED PANELS BY PROGRAM REFIN

### 3.1 Introduction

If a finite element model is refined nonuniformly, more nodes may be generated on one side of some element interfaces within the original model than on the other side. Nodes not joined to nodes in contiguous elements across such an interface are referred to as irregular nodes. The presence of irregular nodes on an element interface implies lack of displacement compatibility unless displacements at all irregular nodes are related to displacements at regular nodes on such an interface.

Constraint equations are typically of the form:

$$u_{dm} = \sum_{n=1}^{n_t} c_{mn} u_{in}$$

where  $u_{dm}$  is the  $n$ -th dependent freedom (either translational displacement or rotation component),  $u_{in}$  are the independent freedoms on which they depend, and  $c_{mn}$  are coefficients relating them. For almost all elements in the VAST library, degree of freedom  $i$  at the dependent node  $m$  is related to only degree of freedom  $i$  at two or more independent nodes  $n$ . What this means for a refinement program such as REFIN is that constraint equation coefficients need be stored for only the first degree-of-freedom at each dependent node. At the end of the program REFIN when multi-point constraint data is being written to the USE file for the refined model, constraint equations for all degrees of freedom at dependent nodes can be created by simple repetition. The plate bending element and the general beam stiffener element however are exceptions. Out-of-plane displacements for points along an edge are cubic functions in a linear edge variable (defined by values of out-of-plane displacement

and rotations at end points). The present note outlines the required modifications to generalize constraint generation algorithms within program REFINE.

A limited capability for the generation of cubic multi-point constraint equations for the triangular plate bending element and the general beam stiffener elements has now been implemented and tested in program REFINE. It permits nonuniform refinements of stiffened or unstiffened plate panels. The panel must be flat and its normal aligned with one of the coordinate axes. The user is prompted for the orientation of the normal. Although it is relatively straightforward to compute the direction of the normal for a stiffened panel, the program nonetheless prompts the user to manually define its direction cosines. However, it is felt that in faceted plate structures, a single normal may have to be utilized for correct generation and manipulations of the multi-point constraint equations.

What distinguishes constraint equation generation for the plate bending and general beam stiffener element types from others in the VAST library is that typically a different constraint equation is required on each freedom at irregular nodes of a nonuniformly refined model. Development of this capability thus involved two principal steps. The first step consisted of the generalization of procedures with REFINE so that more than one constraint equation could be generated and manipulated for each irregular (constrained) node. The second step, of course, was to implement the appropriate cubic constraint equations required for out-of-plane displacements in the plate bending elements. Similar approach is necessary for the general beam stiffener element.

In the following sections of this note these steps are elaborated on and some program usage restriction and guidelines are also discussed. The future development of this work is also discussed briefly.

### 3.2 Generalization of Constraint Equation Manipulation

The basic logic implemented within program REFINe for generation, manipulation, and optional elimination of constraint equations (as well as for computation of new nodal coordinates, and for generation of connectivities for new elements) is adequate for processing of cubic constraint equations. The same nodes are related in each constraint equation associated with a given irregular node; different degrees of freedom are generally involved in each constraint equation and frequently more than one degree of freedom is used at every dependent node. Dependent (irregular) nodes for which displacements are defined via constraint equations are stored in array NK and associated independent nodes in the matrix NJ. Enough information can be extracted from the structure of the first constraint equation at every dependent node for the purpose of filling arrays NK and NJ.

It is primarily in the generation and handling of the coefficients for the constraint equation that implementation of cubic constraint equations for the plate bending elements and general beam stiffener element differs from the implementation of appropriate constraint equations for other elements in the VAST library. In this section, the question of modifications to the constraint equation handling is discussed.

The simplest method of extending the program logic to handle multiple constraint equations where conceivably a different constraint equation is associated with each degree of freedom at a dependent node, is to leave the basic program logic related to determining the status of nodes (regular or irregular), to keeping track of all independent nodes associated with each dependent one etc. intact and setup an auxiliary mechanism (data management system) for storage and retrieval of constraint equation coefficient data. Whenever constraint equation manipulations are required, instead of one set of equations being operated on several sets will be. Generally the number of constraint



equations to be operated on will be equal to the number of nodal degrees of freedom at each irregular node. Of course, repeating constraint equation computations when all constraint equations associated with an irregular node are equivalent is unnecessary and future refinements of this algorithm could conceivably involve investigating the possibility of performing one set of constraint equation calculations when they are equivalent for each degree of freedom at irregular nodes. For such nodes of course, it will be necessary to expand out the constraint equations and generate the NDF constraint equations from the single set of constraint equation coefficients generated by the program.

Due to efforts to improve speed of program REFINE during an earlier contract [1], dependent node numbers and associated constraint equation coefficients were stored in program arrays to reduce the amount of program I/O and thereby increase computational efficiency of critical sections of the refinement algorithm. To store coefficients for all constraint equations at each irregular (dependent) node will raise program storage requirements quite dramatically particularly since the maximum number of structural nodes has recently been increased to 2500.

CHAPTER 4  
AUTOMATIC TREATMENT OF BOUNDARY NODES IN SUPERELEMENT REFINEMENT

#### 4.1 Introduction

Substructuring plays a major role in finite element modelling of ship structures, and any finite element model refinement capability would be incomplete if it could not handle substructured finite element models. Earlier versions of the program REFIN were not fully operational on superelements [1]. A number of restrictions were adopted to facilitate producing a preliminary capability to refine superelements which included:

- a. Only level one superelements can be refined.
- b. A substructure may only define one superelement.
- c. Substructures had to be refined in ascending order.
- d. All new superelement interface nodes had to become either master nodes or slave nodes as defined by an interactive prompt.
- e. Limitations on refinements of previously refined structures are permitted, notably, each element of the original model could be refined only once.

Considerable effort in this contract has been concentrated on removing the above mentioned restrictions. This chapter describes work on Item d.

#### 4.2 Importance of Automatic Classification of Boundary Nodes

Geometrically, refinement of substructured models is no different than refinement of unstructured models. Basically element refinement

still involves the generation of new nodes, removal of the connectivity for the parent element, the generation of connectivities for new elements produced by element refinement as well as generation and manipulations of the constraint equations for irregular nodes on the interface between refined and unrefined elements.

Some auxiliary issues related to substructures alone cannot be avoided when they are refined. Firstly, constraint equations are implemented by the slave node concept whereby dependent nodes for constraint equations are master nodes. Consequently, nodes internal to substructures may have to become master nodes in order that constraint equations can be applied. Secondly, assignment of constraint equations for boundary nodes in one substructure (superelement) cannot be done independently of other substructures of the model. If all substructures sharing a given interface are refined consistently all new nodes generated on the interface can become master nodes. Otherwise, those new nodes which do not have counterparts in all connecting substructures must be slaved (have constraint equations retained for them). A limited capability to refine substructure/superelements was implemented in the previous contract [1]. The approach adopted was to prompt the user whether all boundary nodes were to be identified slave or master nodes. The user was thus responsible for ensuring that the refinement of all substructures/superelements were performed appropriately for the selection made. Figure 4.1 illustrates the two modelling options for new superelement interface nodes.

The restrictions adopted for the initial substructure/superelement refinement capability developed partly from time and budgetary constraints. At the time, it also seemed that the implementation of a general approach for automatically identifying a boundary node to be a master node or slave node would require searching all element connectivities of all superelements within the structure. The large amounts of file searching implied significant increases in CPU requirements and

decreased algorithm (program) speed when substructuring was involved. In fact, recent developments in subroutine MPC lead to the proposal that the approach taken to determine when a constrained node can be eliminated could also be used to determine when a constrained boundary node can be eliminated. If the constraints on a boundary node remained, the node would be identified as a slave node, and if the constraints are removed, the node would be identified as a master node.

The main procedures employed in subroutine MPC to determine whether constraints on nodes could be eliminated required a search through all the element connectivities. The extension of this procedure to a substructured model would thus require a search through all the element connectivities of all the superelements. At the start of this contract, a review of the data requirements for determining whether a boundary node should be specified as a slave or master node revealed that sufficient data for a reliable decision could, in fact, be obtained from the superelement data section of the input file. This data provides the master node list for each superelement and thus defines the connectivity of each superelement. A search through this data would eliminate the need for searching through all the individual elements in the structure. The implementation of this procedure within a new subroutine called MPCSE is detailed in this chapter. The overall structure of the REFIN program does not change beyond the addition of a new module as is readily appreciated by examining Figure 4.2.

The capability to refine substructure/superelements has been improved in this contract to automatically identify the boundary nodes as slave or master. A boundary node is defined as a node located on the perimeter of a superelement.

This chapter will review the steps taken to identify nodes requiring multipoint constraint data. Constraint equations for substructures are implemented by using the constrained substructure option of VAST by

defining independent nodes as master nodes and dependent nodes as slave nodes. The data for constrained node is manipulated on scratch files using the stiffness modification format and is re-formatted to the super-element format only in the final step. From the point of view of geometry an individual substructure is the same as an unstructured model. The treatment of the multipoint constraint data as outlined above allows the REFINE program to operate basically the same way for both a substructured and an unstructured model. The only difference is that for a substructured model the constraint equations are again reviewed after all the substructures have been refined.

#### 4.3 Review of Important Terminology

The following definitions will be useful when discussing constraint equation generation and manipulation. The definitions assigned will be used throughout this chapter.

<u>Border Node</u>	A node which is located on the perimeter of a parent element.
<u>Boundary Node</u>	A node which is located in the perimeter of a superelement.
<u>Dependent Node</u>	A node requiring a multipoint constraint equation.
<u>Interface Node</u>	A border node common to two or more elements.
<u>Internal Node</u>	A node which is not located in the perimeter of a superelement.

#### 4.4 Identification of Node Requiring Multipoint Constraint Equations

Multipoint constraint equations are required for irregular nodes created by non-uniform mesh refinement. The REFINE program generates the

constraint equations for all "border" nodes created as the result of an element refinement and then may remove the constraint through a series of checks performed at three stages before the generation of the final output file. The constrained node is investigated to ensure the constraint is required by subroutine REFIN, subroutine MPC, and if the model is substructured, subroutine MPCSE. The REFIN subroutine is called as each individual element is refined. Subroutine MPC is called after all the elements in a substructure have been refined and MPCSE is called after all the substructures have been refined. The checks performed by each of the subroutines are described in detail in the following sections.

#### 4.5 Constraint Equation Operations in Subroutine REFIN

Subroutine REFIN generates the new element connectivities for the elements created by the refinement of an element and also generates and investigates the multipoint constraint equations. The constraint equations are calculated by subroutines ELEML, ELEMQ, ELEM, or ELEM depending upon the element type. Subroutine REFIN does not generate or investigate those nodes that are completely internal to the parent element and generates constraints only for new nodes (not previously in the nodal coordinate array). However, if a node is a border node and is not new it is checked to see if it is already specified as a dependent node. If it is a dependent node then the equation is checked to ensure all the nodes are included in the connectivity of the element being refined. If the nodes are included then the equation is investigated further to determine if the element being refined is the only element defining the independent basis (for the constraint equation). This operation is performed by calling subroutine ELEM with the ISW parameter set to three. The constraint equation is checked against all the original element connectivities. If the element being refined is the only element defining the independent basis then the constraint is removed. Otherwise the node is an "interface" node and this is indicated by filling the second position of the NIDO array with the number of elements containing the inde-

pendent basis in their connectivity. The identification of the nodes as an "interface" node is required for subroutine MPC.

#### 4.6 Constraint Equation Operations in Subroutine MPC

Subroutine MPC determines which multipoint constraint equations are required and eliminates those which are not. The procedure to determine whether a constraint can be removed is described as follows:

1. All the elements connectivities (if superelements are present, then the connectivities for substructure being refined) are read. After each connectivity is read, the independent nodes of each multipoint constraint equation are compared against the connectivity nodes for all elements. If all independent nodes are present in any connectivity list, the equation must be retained. Otherwise, further investigation of the multipoint constraint equation is necessary before it can be removed.
2. If all independent nodes of any multipoint constraint equation are not contained in any element connectivity, the equation is checked to ensure its independent nodes are not in fact dependent (as a result of constraint equations considered previously). If dependent nodes are found and the node is not an "interface" node then the constraint equation remains. If not, and the geometric data is also not substructured, the constraint is marked to be removed.
3. This step is performed only if the file is substructured. The independent nodes are checked to see if they correspond to master nodes and if the master nodes are found only in the substructure being refined, then the constraint is removed. If the nodes do not correspond to master nodes and the node is marked as an interface node then the constraint is also removed. This step also serves to identify "boundary" nodes which are required for subroutine MPCSE

since the constraint may still yet be removed depending upon the refinement of other substructures.

After the above steps are performed for an unsubstructured model, the constraints which remain are condensed and skew effects are taken into account. The resulting final data is written to the output SMD file.

For a substructured file, additional steps are performed and the file data is not condensed. The "boundary" nodes identified in step 3 have their coordinate locations compared to the existing list of boundary nodes to determine their boundary node number. This procedure is performed by subroutine IDBLIS. The master node array for the substructure being refined is then filled with the negative "boundary" node number. Next, the independent nodes are compared to the master node list and if they are not present then the list is incremented. In the case of a boundary node, the boundary node number and the master node number it is dependent upon are written to the NBOR file for processing in subroutine MPCSE. The data for all the constrained nodes is written to the NREB file.

#### 4.7 Constraint Equation Operations in Subroutine MPCSE

After the refinement of all elements in all substructures has been completed, subroutine MPCSE is called. At this point, all the multipoint constraint equations have been computed and stored on the NREB file. These equations are adjusted to take into effect skew conditions but are not condensed. This subroutine makes use of existing array space. The steps to determine whether the constraint is to remain are as follows:

1. A loop over the superelements is performed to read the data on the NBOR file and fill the NJ array with the master node numbers that the boundary node is dependent upon. The NBOR file was generated in subroutine MPC and has the format given in Table 4.1. The subestruc-



ture and master node numbers for each substructure are written to a file and a second file containing the boundary node numbers is created.

2. A loop over the superelements is again performed to read the master node numbers in core and compare them against the array generated in step 1 to determine if the superelement contains all the master nodes that the boundary node depends upon. If YES, then another array, IREC, is filled with the superelement number and the counter for the number of superelements containing the independent basis of the equation is incremented.
3. The superelements are again looped over and the boundary nodes are now placed in core and compared against the array generated in step 2. The constraint remains on a node if the independent basis is contained in the superelement as flagged in the IREC array but the superelement does not contain the boundary node as determined by the superelements boundary node list. The one exception is if the independent basis is found only in the superelement then it is further investigated to determine if the node is an interior node. If the node is an interior node, it is neither a slave or master node.
4. New master node numbers are assigned to all the border nodes flagged as having their constraint removed.
5. The superelements are again looped over and the following steps are performed:
  - (a) The boundary node numbers flagged as negatives in the master node list for each superelement are substituted with the new master node number calculated in Step 4.

- (b) The multipoint constraint data generated by subroutine MPC is read and the data is checked to see which constraints may be removed. If the node was determined to be a master node, then the constraint is removed at this point.
- (c) The multipoint constraint equations to remain are condensed.
- (d) The data is transformed from the ISTIFM data format to the IELEMB format and written to the output file.

#### 4.8 Sample Problems

In this section, the results of refining an unsubstructured model is first presented to demonstrate how the REFIN program determines which nodes to have multi-point constraint equations applied. A substructured model is then refined to illustrate the treatment of boundary nodes.

##### Unsubstructured Model Problem

The unrefined model is a square plate composed of nine 4-noded quadrilateral shell elements (IEC=5) arranged in a 3x3 mesh. The central element is first refined to order 2 and then two of the resulting elements are refined again to order 2. The resulting mesh is shown in Figure 4.3 and the following discussion will refer to it.

The refinement of the central element results in new nodes 17 through 21. The border nodes 17, 18, 20 and 21 are specified as constrained in subroutine REFIN.

The refinement of the first new element (with corner nodes 6, 17, 19, 18) results in new nodes 22 through 26. The border nodes 22, 23, 25 and 26 are specified as constrained in subroutine REFIN.

The refinement of the second new element (with corner nodes 17, 7, 20, 19) results in new nodes 27 through 30. The border nodes 27, 29 and 30 are specified as constrained in Subroutine REFINE. Node 25 is a border node but is not new and is determined to have been already specified as a constrained node. The node is specified as an "interface" node since its independent basis is contained in more than one element.

The element refinement specifications are complete and now subroutine MPC is used to determine which constraint equations are required. The constraint equations generated are uncondensed which means a constrained node is related to its two adjacent nodes along the edge being refined. Table 4.2 identifies the independent nodes for each dependent node for the uncondensed equations of this sample problem. Constrained nodes 17, 18, 20 and 21 remain since the comparison against the connectivity list revealed that the independent nodal basis for each is contained in the connectivity of at least one element in the model. Constrained nodes 22, 23, 26, 27, 29 and 30 also remain since the independent nodal basis of each contains dependent nodes which are not interface nodes. The constraint equations on node 25 were removed at this point since the node was flagged as an interface node.

The constrained nodes to remain are condensed and the resulting data is written to the input file.

#### Substructured Model Problem

The unrefined model is a ship transverse bulkhead composed of four structures/superelements shown in Figure 4.4. Both superelements 1 and 2 will be partially refined along their interface and the resulting mesh is shown in Figure 4.5. VASTG graphics has no provision for identification of constrained nodes (slave nodes) for substructures. It is therefore not possible to demonstrate the operation of REFINE for substructures using graphics only and necessary reference will be made to the super-element data (PREFIX.SED).

The superelement data for the unrefined model is shown in Table 4.2. Superelement 1 has 64 master nodes which are plotted in Figure 4.6 using the option to plot border nodes labelled with local node numbers and superelement 2 has 80 nodes which are plotted in Figure 4.7. The refinement of superelements 1 and 2 result in the generation of models with nodes as shown in Figures 4.8 and 4.9. The new nodes generated internally to the refined mesh create no changes to the number of slave or master nodes. Table 4.5 presents the refined superelement data for the model. The ten nodes in each refined superelement which are identified as slave nodes in this data have been labelled selectively as shown in Figures 4.10 and 4.11. The two new nodes located along the interface between superelements 1 and 4, namely nodes 160 and 162, remain constrained as do nodes 167 and 146 located between superelements 2 and 3 since superelements 3 and 4 are not refined. These four nodes are interface nodes and are marked as negative in the restart data produced under the REFINES header in the final output file. Chapter 5.0 contains a description of the reason for this procedure.

Nodes 145 and 161 of superelement 1 and nodes 166 and 147 of superelement 2 are identified as slave nodes. These nodes would become master nodes if boundary conditions were applied to them. The remainder of the nodes labelled as slave nodes are internal to each substructure and are constrained since their dependent basis is contained in another element. The new master nodes which are identified as a result of the refinement are the new nodes located along the interface between superelements 1 and 2 and also the four existing nodes required to be identified as master nodes for each refined superelement because these nodes were identified in the dependent basis of a constrained nodes.

TABLE 4.1

Format of File NBOR Generated in Subroutine MPC

NSS	NBORD	NGN,NBORD,NDDOF,IDCNK,XO,YO,ZO
		BORD NO., NMNP, (MMM(I),I=1,NMNP)
		where: MMM contains master node numbers the boundary node is dependent upon
		NSUB,NMN,NBORD
		(NMN1(I),I=1,NMN)
		(NMN2(I),I=1,NMN)
		(SPACE(I),I=1,NBORD)
		where: NMN1 contains substructure node numbers NMN2 contains master node numbers and boundary nodes are input as negative number SPACE contains boundary node numbers

TABLE 4.2

Uncondensed MPC for Sample Problem

Dependent Nodes in MPC Equation	Independent Nodes in MPC Equation
17	6,7
18	6,10
20	7,11
21	10,11
22	6,17
23	6,18
25	17,19
26	18,19
27	7,17
29	7,20
30	19,20

Superelement Data for Unrefined Bulkhead Model

4	1	64	0	OCPF MAJOR BKHD AT STATION 34.0 BKHD 269,270 & 271 COMPLETES SHAPE															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
0.000	0.000	0.000																	
2	80	0	OCPF MAJOR BKHD AT STATION 34.0 BKHD 134,270 & 271 COMPLETES SHAPE																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121				
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80				
80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95				
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111				
111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126				
122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137				
0.000	0.000	0.000																	
3	86	0	OCPF MAJOR BKHD AT STATION 34.00 ,BKHD 134,269 & 271 COMPLETES SHAPE																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103				
125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155				
156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171				
172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187				
188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203				
203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218				
0.000	0.000	0.000																	
4	86	0	OCPF BKHD AT STATION 34.0 BKHD 270, 269 & 134 COMPLETES SHAPE																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103				
125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140				
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80				
211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226				
227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242				
242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257				
257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272				
258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273				
0.000	0.000	0.000																	

### Superelement Data for Refined Bulkhead Model

1	74	10														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	
67	72	125	126	127	89	94	134	135	136							
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	
274	275	282	283	284	276	277	285	286	287							
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00												
119	2	6														
36	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.500E+00	-0.125E+03	0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.500E+00	0.125E+03	-0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.150E-02	-0.250E+00	0.750E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	-0.150E-02	-0.250E+00	0.750E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	-0.150E-04	0.750E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.150E-04	0.750E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
36	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
120	2	6														
67	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.500E+00	-0.109E+03	-0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.500E+00	0.109E+03	0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.172E-02	-0.250E+00	-0.862E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	-0.172E-02	-0.250E+00	-0.862E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.198E-04	-0.862E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	-0.198E-04	-0.862E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
121	2	6														
49	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.109E+03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.500E+00	-0.109E+03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	-0.172E-02	-0.250E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.172E-02	-0.250E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
128	2	6														
49	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.500E+00	-0.106E+03	0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.000E+00	0.500E+00	0.106E+03	-0.125E+01	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	0.000E+00	-0.250E+00	0.882E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.000E+00	-0.175E-02	-0.250E+00	0.882E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
49	0.000E+00	0.000E+00	-0.208E-04	0.882E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.000E+00	0.208E-04	0.882E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00								



89	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
99	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
99	0.000E+00	0.000E+00	0.500E+00	-0.987E+02	0.250E+01	0.000E+00
94	0.000E+00	0.000E+00	0.500E+00	0.987E+02	-0.250E+01	0.000E+00
89	0.000E+00	0.000E+00	0.190E-02	-0.250E+00	0.190E-01	0.000E+00
94	0.000E+00	0.000E+00	-0.190E-02	-0.250E+00	0.190E-01	0.000E+00
89	0.000E+00	0.000E+00	-0.480E-04	0.190E-01	0.500E+00	0.000E+00
94	0.000E+00	0.000E+00	0.480E-04	0.190E-01	0.500E+00	0.000E+00
89	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
100	2	6				
12	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.500E+00	0.101E+03	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.500E+00	-0.101E+03	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	-0.185E-02	-0.250E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.185E-02	-0.250E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
145	2	6				
37	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.700E+02	0.000E+00
38	0.000E+00	0.000E+00	0.500E+00	0.000E+00	-0.700E+02	0.000E+00
37	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00
38	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	-0.268E-02	0.000E+00	-0.250E+00	0.000E+00
38	0.000E+00	0.000E+00	0.268E-02	0.000E+00	-0.250E+00	0.000E+00
37	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
160	2	6				
13	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.500E+00	0.125E+01	0.674E+02	0.000E+00
14	0.000E+00	0.000E+00	0.500E+00	-0.125E+01	-0.674E+02	

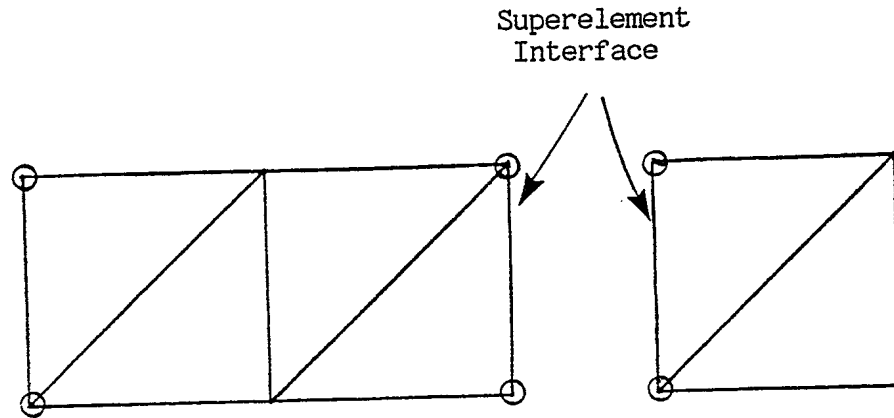
TABLE 4.4 (Continued)

2	91	10																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48				
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64				
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121				
67	72	89	94	144	145	160	162	163	164	165									
65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80				
80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95				
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111				
111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126				
122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137				
278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293				
0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
124	2	6																	
36	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
36	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
36	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
36	0.000E+00	0.000E+00	0.150E-02	0.250E+00	0.750E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.150E-02	0.250E+00	0.750E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
36	0.000E+00	0.000E+00	0.150E-04	0.750E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.150E-04	0.750E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
36	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
125	2	6																	
67	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.172E-02	0.250E+00	0.862E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.172E-02	0.250E+00	0.862E-02	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.198E-04	0.862E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.198E-04	0.862E-02	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
67	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
126	2	6																	
49	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.109E+03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.109E+03	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.172E-02	0.250E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.172E-02	0.250E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
72	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
130	2	6																	
49	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
89	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
89	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00				
49	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.000E+00														

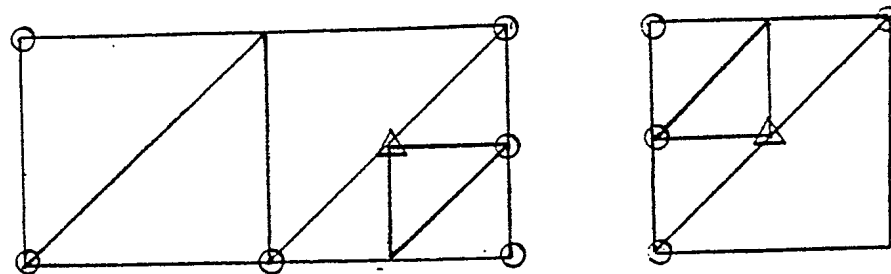
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94	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
89	0.000E+00	0.000E+00	0.500E+00	0.987E+02	0.250E+01	0.000E+00
94	0.000E+00	0.000E+00	0.500E+00	0.987E+02	0.250E+01	0.000E+00
89	0.000E+00	0.000E+00	0.190E-02	0.250E+00	0.190E-01	0.000E+00
94	0.000E+00	0.000E+00	0.190E-02	0.250E+00	0.190E-01	0.000E+00
89	0.000E+00	0.000E+00	0.480E-04	0.190E-01	0.500E+00	0.000E+00
94	0.000E+00	0.000E+00	0.480E-04	0.190E-01	0.500E+00	0.000E+00
89	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
132	2	5				
12	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.500E+00	0.101E+03	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.500E+00	0.101E+03	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.185E-02	0.250E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.185E-02	0.250E+00	0.000E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.500E+00	0.000E+00
12	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
94	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
145	2	5				
13	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
13	0.000E+00	0.000E+00	0.500E+00	0.125E+01	0.694E+02	0.000E+00
14	0.000E+00	0.000E+00	0.500E+00	0.125E+01	0.694E+02	0.000E+00
13	0.000E+00	0.000E+00	0.487E-04	0.500E+00	0.135E-01	0.000E+00
14	0.000E+00	0.000E+00	0.487E-04	0.500E+00	0.135E-01	0.000E+00
13	0.000E+00	0.000E+00	0.270E-02	0.135E-01	0.250E+00	0.000E+00
14	0.000E+00	0.000E+00	0.270E-02	0.135E-01	0.250E+00	0.000E+00
13	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
14	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
147	2	5				
37	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
38	0.000E+00	0.500E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
37	0.000E+00	0.000E+00	0.500E+00	0.000E+00	0.790E+02	0.000E+00
38	0.000E+00	0.000E+00				

TABLE 4.4 (Continued)

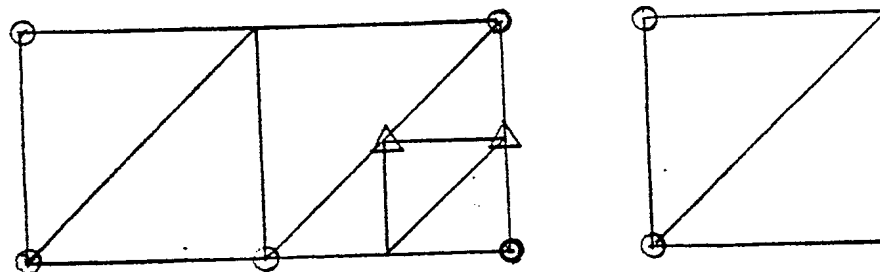
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65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	
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97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	
113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	
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593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	
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625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	
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737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	
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769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	
785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	
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849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	
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897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	
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945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	
961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	
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993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	
1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	
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1281	1282	1283	1284	1285	1286	1287	1288	1289	1290	1291	1292	1293	1294	1295	1296	
1297	1298	1299	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310	1311	1312	
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1345	1346	1347	1348	1349	1350	1351	1352	1353	1354	1355	1356	1357	1358	1359	1360	
1361	1362	1363	1364	1365	1366	1367	1368	1369	1370	1371	1372	1373	1374	1375	1376	
1377	1378	1379	1380	1381	1382	1383	1384	1385	1386	1387	1					



(a) Original Finite Element Model - 2 substructures



(b) Refinement of (a) using the option of all superelement interface nodes as master nodes. Requires adjacent superelements to be refined to same order along common interface.



(c) Refinement of (a) using the option of all superelement interface nodes as slave nodes.

FIGURE 4.1: Modelling Options for New Superelement Interface Nodes  
(O - Master Node,  $\Delta$  - Slave Node)

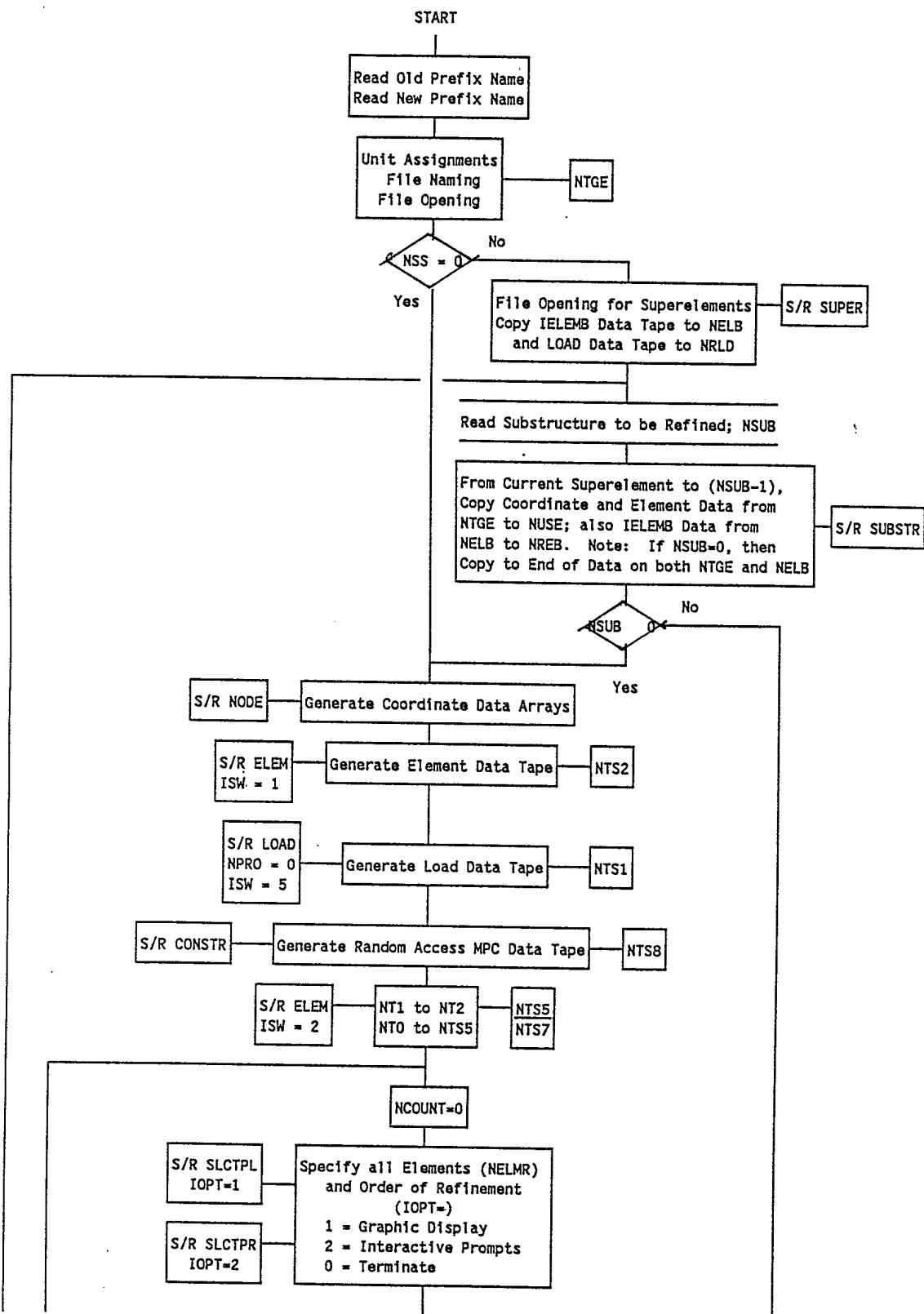


FIGURE 4.2: Flowchart for Program REFINE

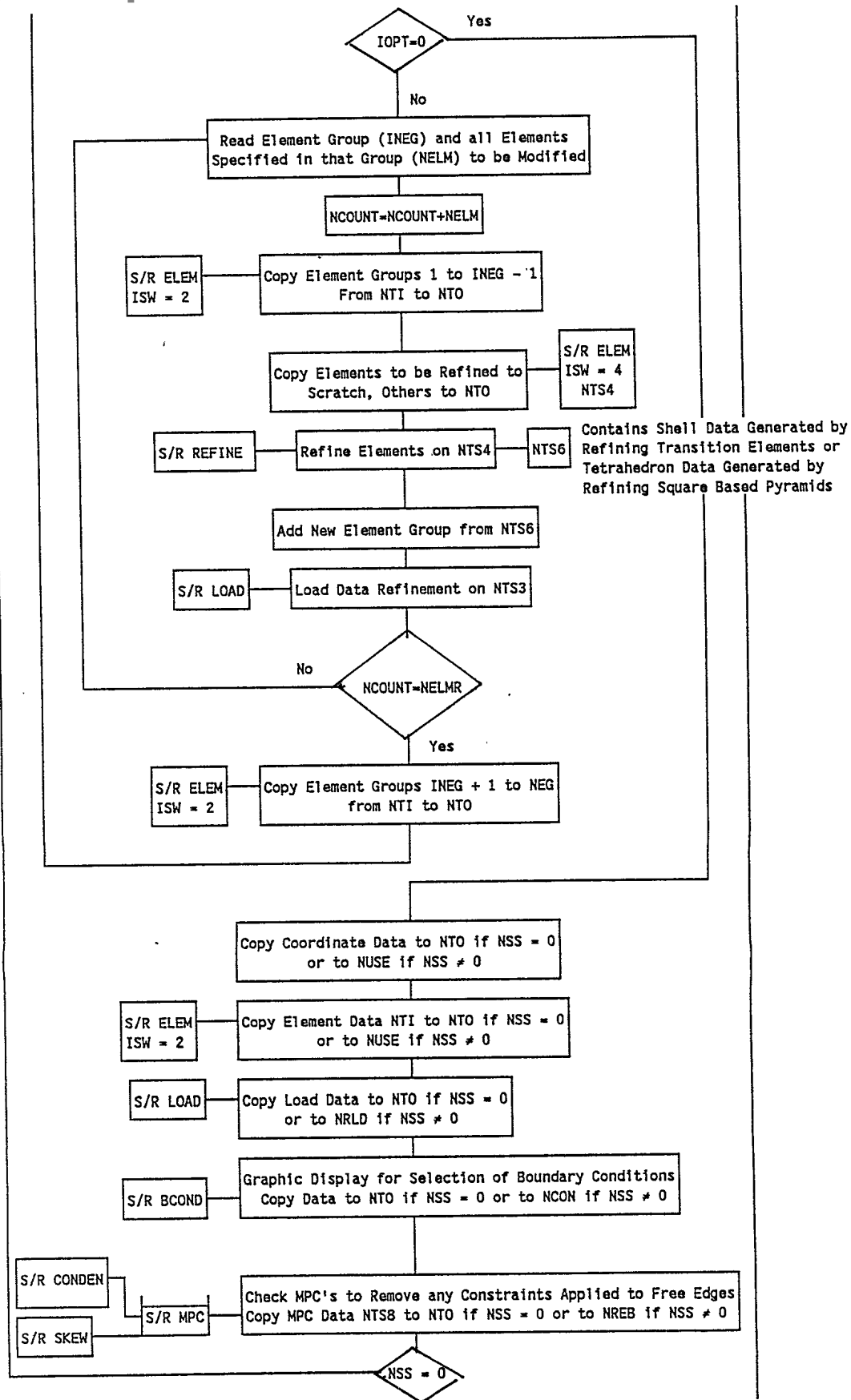


FIGURE 4.2: Flowchart for Program REFINE (Continued)

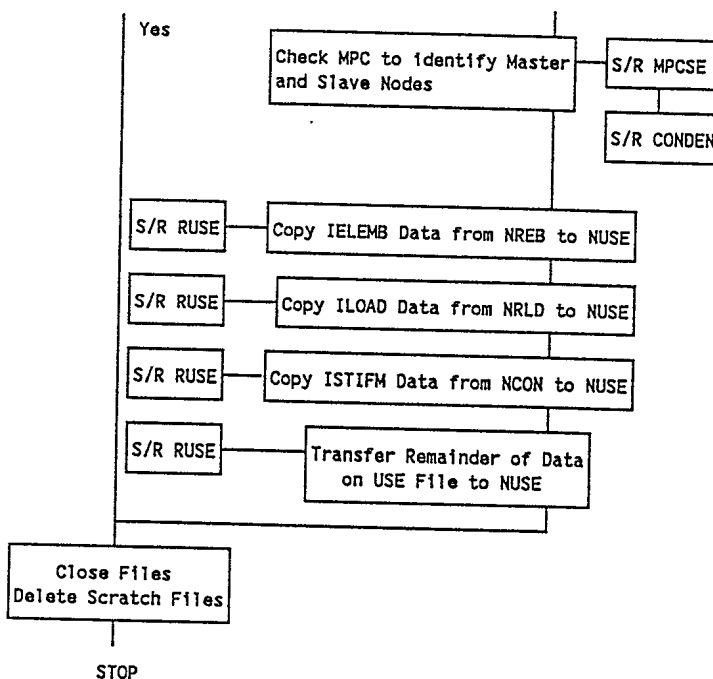


FIGURE 4.2: Flowchart for Program REFINE (Continued)



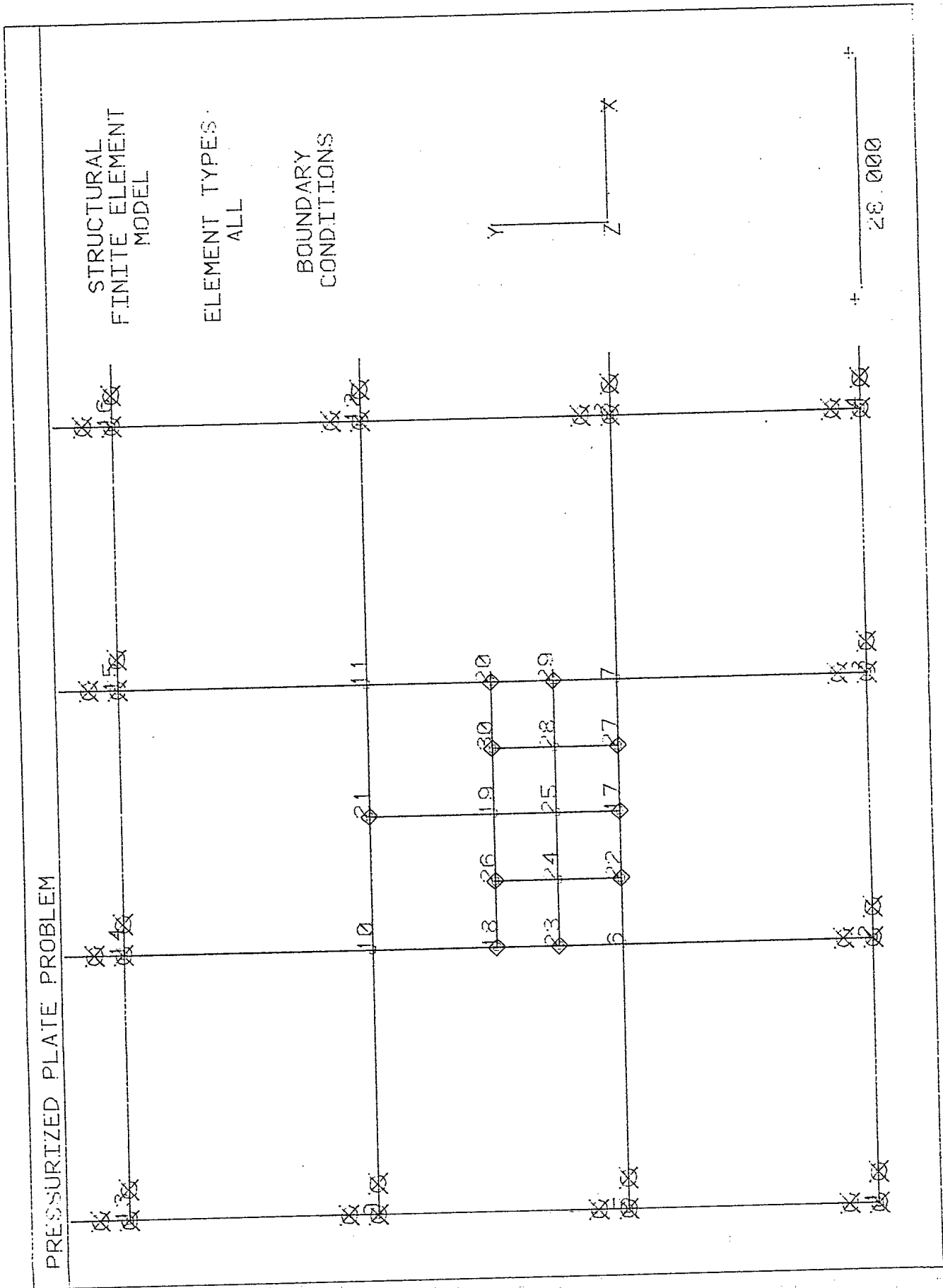


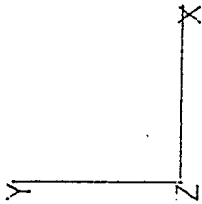
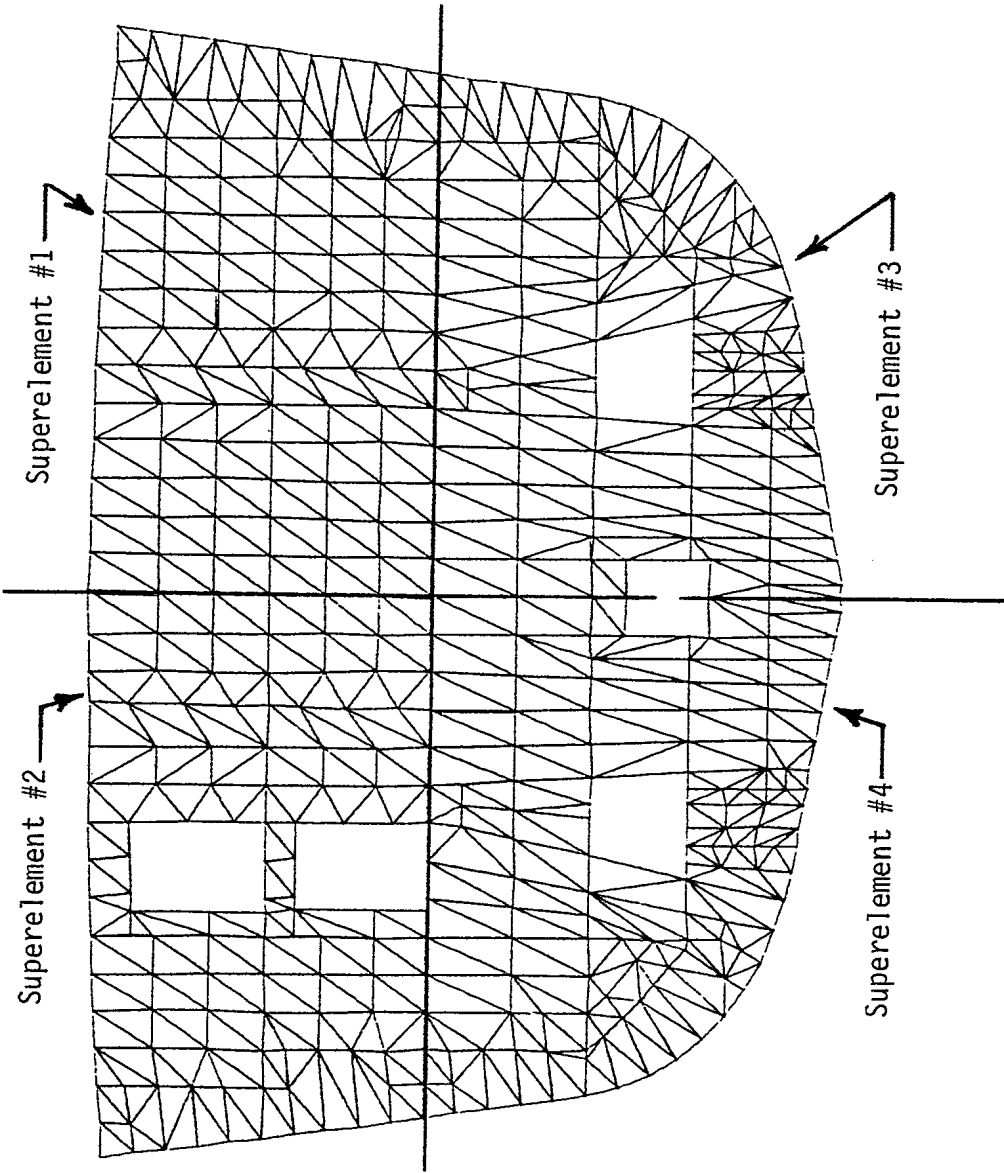
FIGURE 4.3: Recursive Refinement of Square Plate Discretized by 4-Noded Quadrilateral Elements ( $\diamond$  = Dependent Node)

SUBSTRUCTURED BULKHEAD PROBLEM

STRUCTURAL  
FINITE ELEMENT  
MODEL

SUPERELEMENTS  
ALL

ELEMENT TYPES  
ALL



5124.216

FIGURE 4.4: Substructured Bulkhead Model Before Refinement

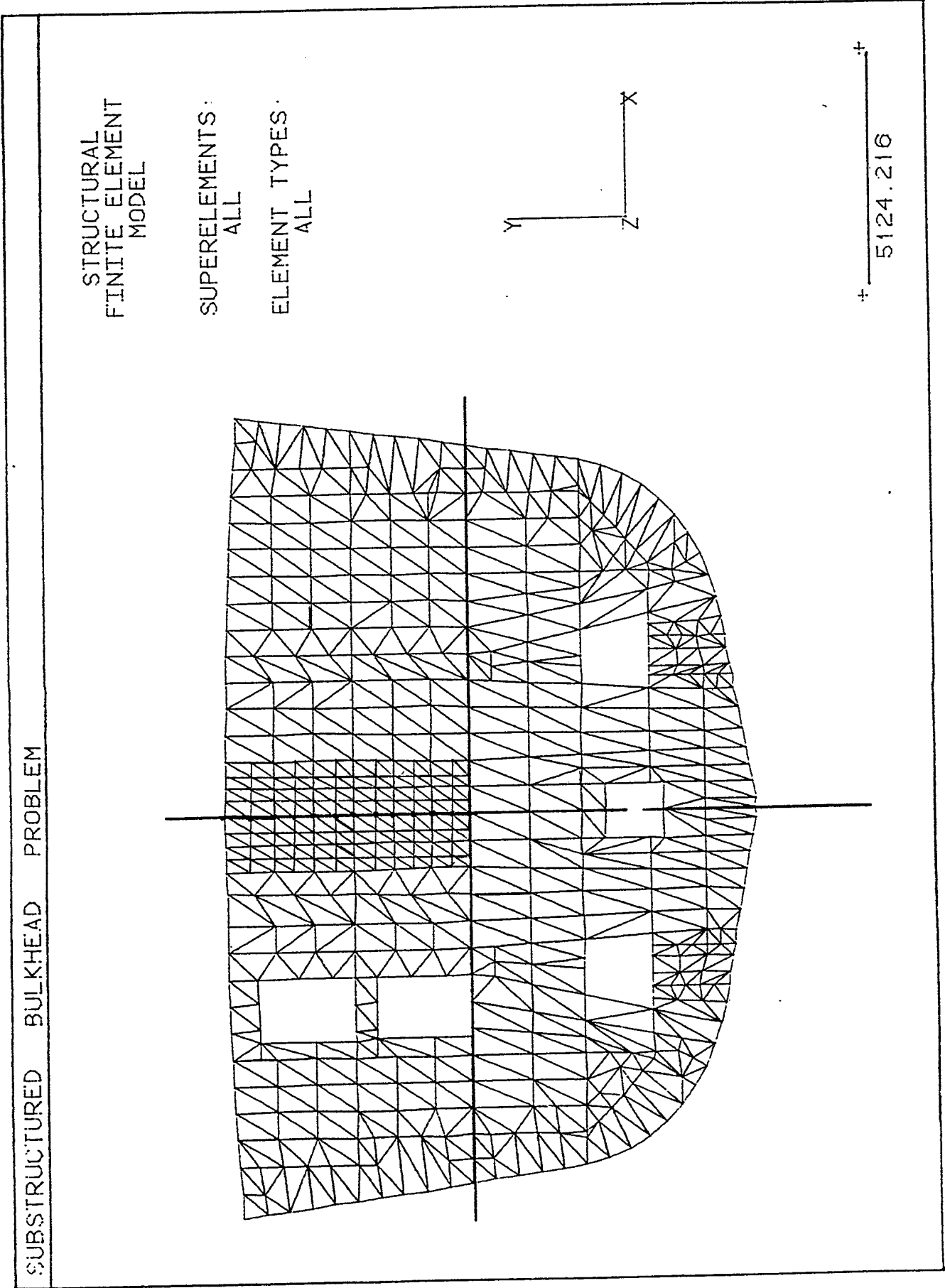
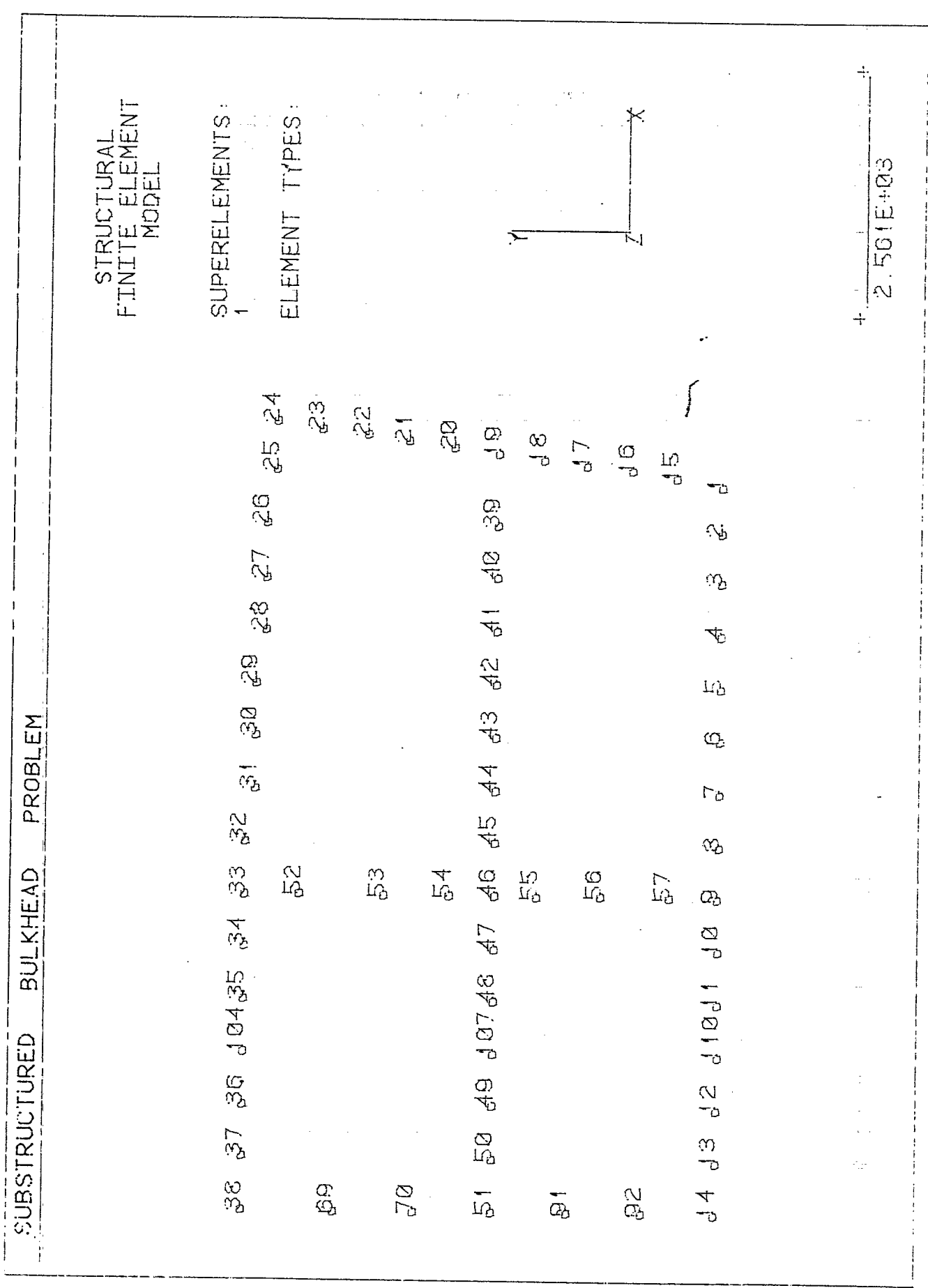


FIGURE 4.5: Substructured Bulkhead Model After Refinement



+ 2.501E+03

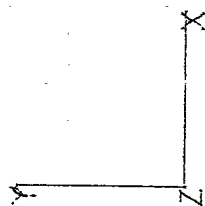
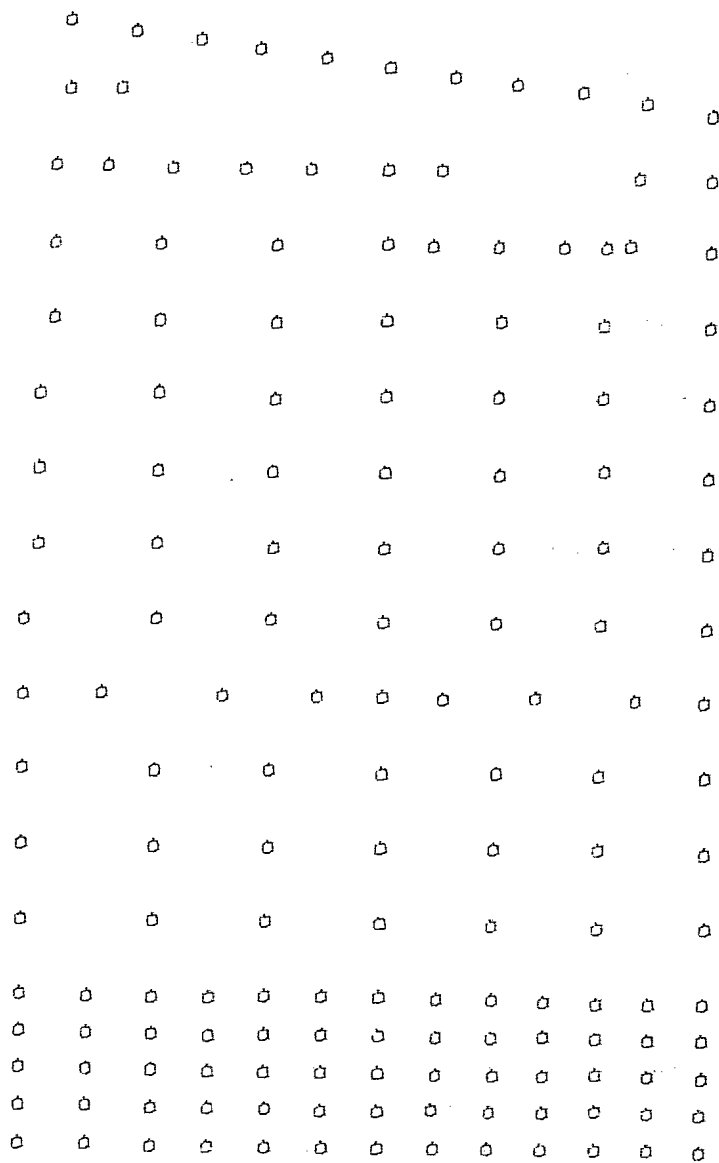
FIGURE 4.6: Master Nodes for First Superelement

SUBSTRUCTURED				BULKHEAD				PROBLEM				STRUCTURAL FINITE ELEMENT MODEL				SUPERELEMENTS 2				ELEMENT TYPES			
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38									
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
19	39	40	41	42	43	44	45	46	47	48	49	50	51										
18	0	0	0	0	0	0	0	0	0	0	0	0	0										
17	0	0	0	0	0	0	0	0	0	0	0	0	0										
16	0	0	0	0	0	0	0	0	0	0	0	0	0										
15	0	0	0	0	0	0	0	0	0	0	0	0	0										
14	2	3	4	5	6	7	8	9	10	11	12	13	14										
<div><div></div><div></div><div></div></div>																							
2.575E+03																							

SUBSTRUCTURED    BULKHEAD    PROBLEM

STRUCTURAL  
FINITE ELEMENT  
MODEL

ELEMENT TYPES:



+ 2551.104

FIGURE 4.8: Nodes for First Superelement After Refinement

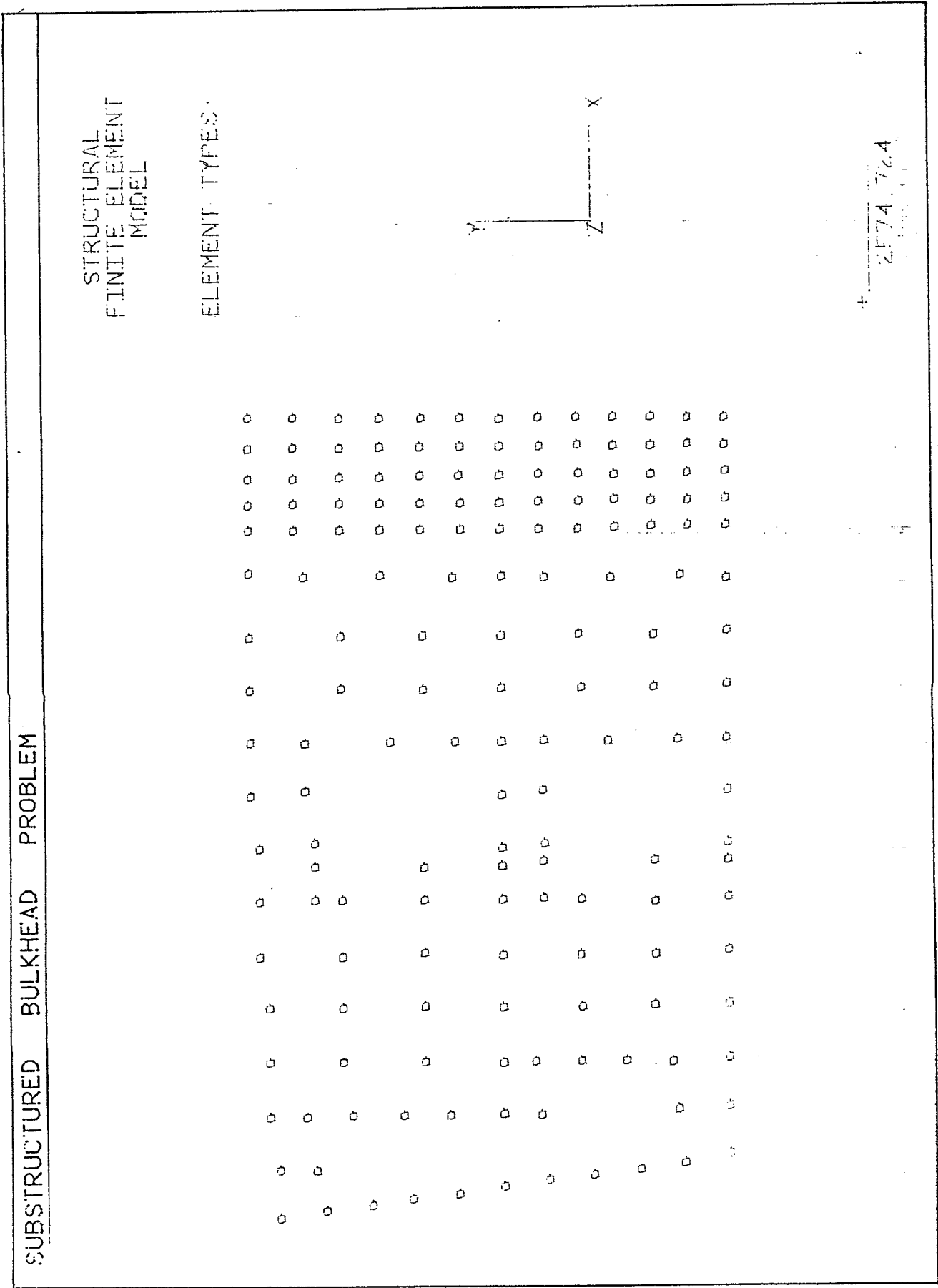


FIGURE 4.9: Nodes for Second Superelement After Refinement

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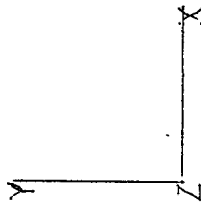
$$\frac{1.039E+03}{+}$$


FIGURE 4.10: Slave Nodes for First Superelement Defined Interactively



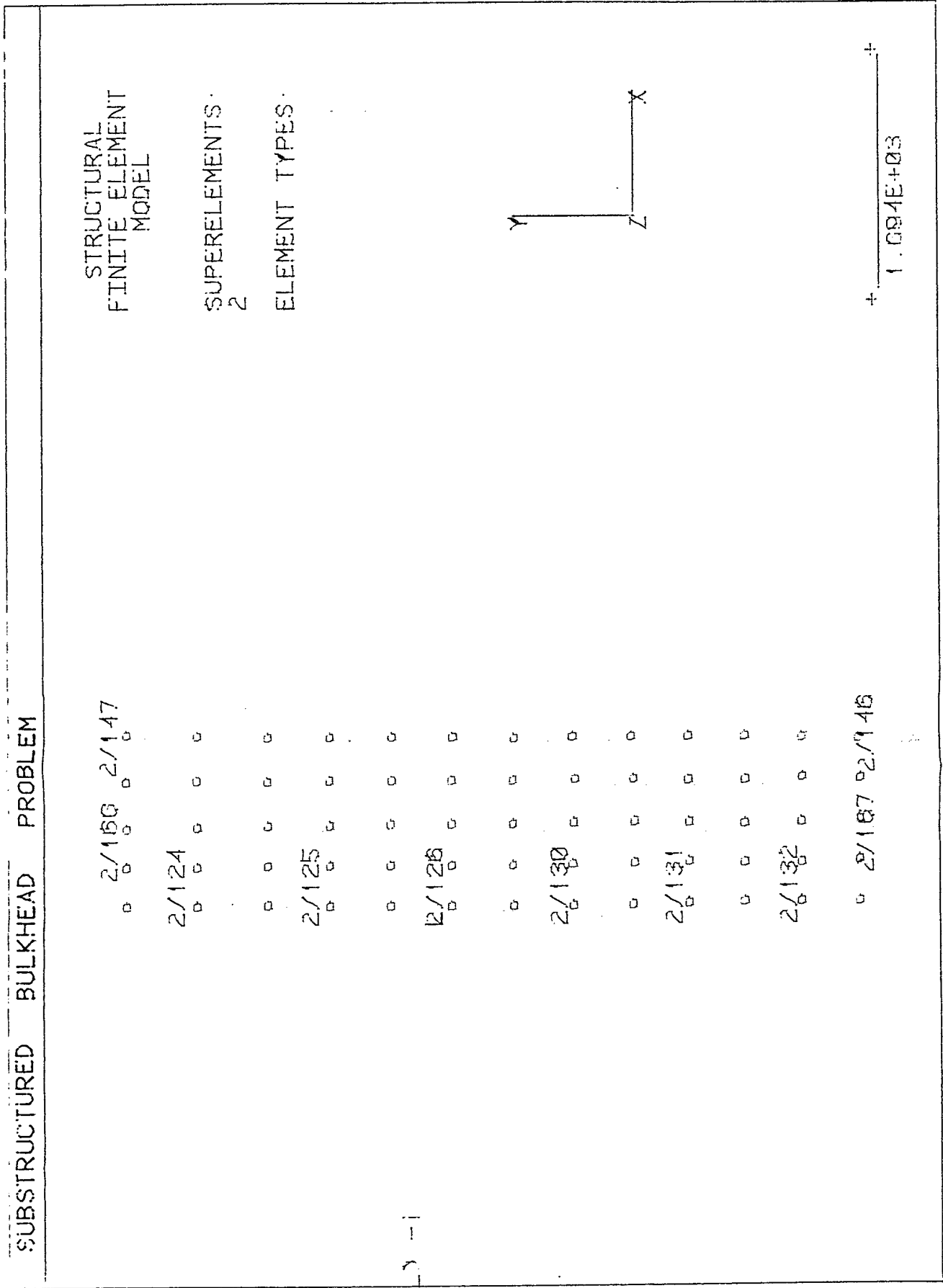


FIGURE 4.11: Slave Nodes for Second Superelement Defined Interactively

## CHAPTER 5

### RESTART CAPABILITY

#### 5.1 Introduction

The REFIN program may eventually be automated to the point that it could operate as a component of an adaptive finite element model optimization capability which once initiated would, without any user intervention, alternately refine the mesh and resolve until desired levels of accuracy are attained with near optimal distribution of freedoms in the finite element model. But considerable further development of the algorithms within both the discretization error estimation and mesh refinement capabilities, is still required. The capability to refine a previously refined structure provides a so called "restart" feature and moves a step closer to the completely automated procedure.

Prior to the development of this capability, the user had to reconsider the original model and refine the model to increasing levels as desired. Now the user has only to "restart" and refine the previously refined model in areas of interest. Considerable savings in time are possible since the user no longer has to reconstruct the refinement process to attain the starting point from which to perform the desired refinements. The implementation of this capability is discussed in this chapter.

#### 5.2 Implementation

The refinement of an element involves:

- 1) generation of new nodes;
- 2) replacement of the connectivity of a parent element with the connectivities of elements produced by the refinement; and
- 3) generation and manipulation of any resulting constraint equations.

When restarting the refinement of a finite element model, no particular problems are expected with the first two steps but difficulties may arise with the manipulation of constraint equations. The algorithm which determines whether constraint equations remaining on each new node requires that constraint equations be available in the uncondensed form. This implies that the constraint equations for the original model must be available in uncondensed form. The algorithm for evaluating whether constraint equations remain on a node is explained in detail in Chapter 4.

The uncondensed multipoint constraint data for a refined model cannot be readily extracted from the condensed constraint equations data written to the stiffness modification data section of the input file. The key to restarting the refinement of a finite element model is just simply to store the uncondensed constraint equations for any refined model in case the refinement process is to be restarted. The restart data associated with the refined model data is therefore stored on the USE file under the header REFIN. On restarts, the REFIN program searches for this header and transfers the data to a scratch file prior to any constraint data processing.

Additional data is required when restarting the refinement of a substructured model if the nodes which are boundary nodes are to be identified. Recall that a boundary node is one which is located on the perimeter of a superelement. Boundary nodes are flagged under the REFIN header by a change of sign on the dependent node. A new subroutine RESTART was developed to extract the information about the boundary nodes from the "restart" data and to fill the master node and boundary node arrays. Master node and boundary node data are used in subroutine MPCSE to determine whether a constraint equation on a node is to be retained and the node becomes a slave node or alternately the constraint equations on a node are eliminated and the node becomes a master node.

The format of the data under the REFINe header is the same for each superelement of a substructured model as for an unstructured model. The variable NDDOF equal to the number of degrees of freedom is entered as zero for those superelements which do not contain any slave nodes.

### 5.3 Sample Problem

The sequence of steps involved in the recursive refinement of a simple model composed of only two four-noded shell elements is now described. This example illustrates the difference between "uncondensed" and "condensed" equations and the requirement for the former in the restarted refinement of the model. The original model composed of two four-noded shell elements is shown in Figure 5.1(a).

The first step in the refinement of the model is to refine one element, as shown in Figure 5.1(b), and generate a constraint equation for node c in subroutine REFINe with nodes a and b specified as dependent nodes.

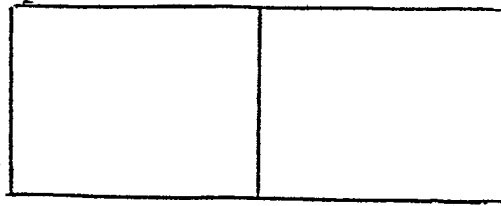
One of the new elements is then refined further as shown in Figure 5.1(c). This results in the creation of constraint equations for nodes e, f, and g. Node e is dependent upon nodes a and c, node f is dependent upon nodes c and d, and node g is dependent upon nodes d and h. Because node c is already a dependent node, the constraint equations for nodes e and f are said to be "nested".

After all specified elements have been refined, the final step dealing with constraint equations involves determining which constraints are to be retained and writing them to the final output file in a format acceptable to VAST. In the example being considered, the constraints on nodes c, f, and g will remain since independent basis of each is contained in connectivity of another element. The constraint on node e also remains since it is dependent upon a dependent node and is not an inter-

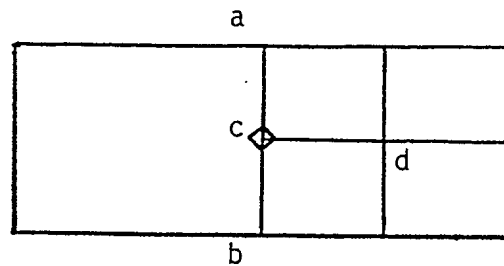
face node. The reader will recall that by the convention established in the introduction to the present chapter, an interface node is defined as a node located on the perimeter of a parent element that is common to two or more elements. The constraint equation for nodes e and f are condensed so that no dependent nodes are involved. The final result is node e dependent upon a and b and node f dependent upon nodes a, b and d.

Figure 5.1(d) shows the result of a restart and in which the second of the original elements is refined. Before the restart capability was implemented in the REFIN program, the constraint on e and f would both be incorrectly removed when the second element was refined. The reason for this is that the constraint equations on node c would be removed in subroutine REFIN since the independent basis for this equation would be determined to be only in the element being refined. The constraints in the remaining nodes would again be investigated by subroutine MPC. The constraint equations on node g would remain since its basis, as in the first model refinement, is contained the connectivity of an element. The constraint equations on nodes e and f are however removed since their independent condensed basis are not in the connectivity of any element in the model and there are no dependent nodes in the equations.

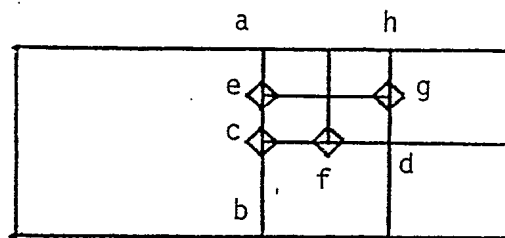
When REFIN is restarted using the uncondensed equations, the constraint equations are manipulated correctly. The constraints on nodes e and f are retained as they should since independent nodal basis is determined to be in an element connectivity.



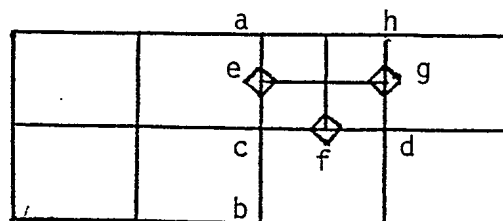
(a) Original mesh with two four-noded shell elements.



(b) Mesh after refining one of the original elements.



(c) Mesh after refining one of the elements generated by refinement.



(d) Mesh after refining second of the original elements.

FIGURE 5.1: Recursive Mesh Refinements and Implication for Dependent Node Determinations  
( $\diamond$ — Dependent Node)



## CHAPTER 6

### OTHER EXTENSIONS OF SUPERELEMENT REFINEMENT CAPABILITIES

#### 6.1 Introduction

Earlier versions of the program REFINe were not fully operational on superelements. A number of restrictions had to be adopted to facilitate producing a preliminary capability to refine superelements. They were identified in Chapter 4. Items d and e were addressed in Chapters 4 and 5, respectively. Items a to c will be discussed in this chapter.

- a. Only level one superelements can be refined.
- b. A substructure may only define one superelement.
- c. Substructures must be refined in ascending order.

#### 6.2 Extension to Multilevel Superelements

The restriction that only level one superelements can be refined still applies. The extension of the program to multilevel superelements would generalize the range of program application but this development appears to be premature at the moment. It is preferred that the superelement capability be thoroughly tested and evaluated before the multilevel superelement capability be introduced.

#### 6.3 Removal of Limitation that One Substructure Defines One Superelement

The removal of the restriction that one substructure may define only one superelement represents the first step in generalizing the refinement algorithm for superelements. Although necessary program modifications are sketched in the following section, they have not as yet been imple-



mented. The time and fiscal constraints of the current contract did not permit this work to be completed.

Of several possible approaches to perform this task that have been investigated, only the two approaches which appear most feasible are discussed in this section.

One requirement arising from the removal of the limitation that one substructure define one superelement is the need for an accounting procedure to determine the number of new substructures created to relate superelements with them. The only information required to determine the number of substructures in a refined model is a list of superelement numbers to be refined. Prompting the user at the start of the program's execution to enter the superelements to be refined provides data which if placed in an array can be used to direct the refinement of the model.

This data is required by subroutine RELEMB which reads the superelement data and now must relate the superelement to the correct substructure numbers. The VAST data input requirement that superelement must be ordered for increasing substructure number limits the options available for programming the superelement refinement. Firstly, the original superelement number scheme can remain unchanged and the substructure numbering changed to effectively create one substructure for each superelement. This has the advantage of allowing the load refinement process to remain unchanged and the refinement to proceed along the input file. Secondly, the superelement numbering may change to exploit any repeated substructures which remain after refinement.

The two options for removal of the limitation that one substructure defines one superelement are best presented by an example. The example to be considered involves a model comprising of 6 superelements generated from 2 substructures for which there is a requirement to refine superelements 2 and 4 (see Table 6.1). The new substructure numbering scheme

resulting from changes in the substructure numbers is shown in Table 6.2. This procedure would result in 6 substructures when 4 would be sufficient since substructures 1, 3, and 5 are the same. The renumbering of the superelements only results in the minimum of 4 substructures as shown in Table 6.3. However, this procedure would require reordering the load data as well as the superelement data. The geometry data for all the substructures to be refined would have to be written to a file so that the refinement process could proceed sequentially along the file.

#### 6.4 Refinement of Superelements in Arbitrary Orders

The restriction that the substructures must be refined in ascending order also still applies. The present algorithm proceeds along the input file with no rewinds which is necessary to keep the I/O at a minimum. The execution time of program REFINER appears to be greatly affected by I/O.

TABLE 6.1

Sample Model with Six Superelements  
Generated from Two Substructures

Superelement Number		Substructure Number	
Old	New	Old	New
1	1	1	1
2*	2	1	2
3	3	1	3
4*	4	1	4
5	5	1	5
6	6	2	6

\* Superelement to be refined.

TABLE 6.2

Superelement Refinement with all Superelements  
Associated with Independent Substructures

Superelement Number		Substructure Number	
Old	New	Old	New
1	1	1	1
2*	2	1	2
3	3	1	3
4*	4	1	4
5	5	1	5
6	6	2	6

\* Superelement to be refined.

TABLE 6.3

Superelement Refinement with Substructure Reordering  
to Minimize Substructures in Model

Superelement Number		Substructure Number	
Old	New	Old	New
1	1	1	1
3	2	1	1
5	3	1	1
6	4	1	2
2*	5	1	3
4*	6	2	4

\* Superelement to be refined.

## CHAPTER 7

### ADDITIONAL DEVELOPMENTS IN PROGRAM REFIN

#### 7.1 Introduction

A number of additional changes to program REFIN, not explicitly called for in the contract statement of work but determined to be desirable during development work and in application of the program, were also implemented. These changes are described briefly in the following sections.

#### 7.2 Option for Data Input from Separate Files

The REFIN program was modified to optionally accept the following data on separate files:

- 1) Geometry Data from file PREFX.GOM
- 2) Superelement Data from file PREFX.SED
- 3) Stiffness Modification Data from file PREFX.SMD
- 4) Load Data from file PREFX.LOD

The REFIN program still places all the output data in the PREFN.USE file and not in separate files.

#### 7.3 Arrays Dimensioning in the Main Module

The changes required to achieve the option for data input from separate files in the REFIN program resulted in some more general organizational changes in the program. For instance, array data transfers within the program were modified so that all arrays are now dimensioned in the main program and passed through the argument lists of the various subroutines. This change facilitates any future increases in array dimensions to accommodate large scale problems. The restrictions adopted for array sizes are as follows:

NGNMAX = 999 (number of geometric nodes)  
NMNMAX = 999 (number of master nodes)  
NTIMAX = 999 (number of time steps)  
NERMAX = 250\* (number of elements to be refined)  
NEGMAX = 250\* (number of element groups)  
NDDMAX = 999 (number of dependent nodes)  
NIDMAX = 8\* (number of independent nodes)  
NDFMAX = 6\* (number of dependent degrees of freedom)  
NSKCMAX = 100 (number of skewed systems)

All of the above array sizes except those marked with an asterisk correspond with VAST program limitations.

#### 7.4 Superelement Load Data Format Change

The REFIN program was modified to accommodate the new format in VAST for load data on superelements which involves the card identifying the number of the superelement to be loaded being preceded by an asterisk (\*). The implied changes for the REFIN program were concentrated in the subroutine RUSE, which reads from the specified input file and optionally writes it to the specified output file according to the value of the ISW parameter. This subroutine previously terminated the data transfer process and returned control to the main program when either a "I", "R" or "\*" was found in the first column. The asterisk previously only identified the beginning of the geometry for a new substructure. The ISW parameter was modified so that "0" indicates that the data is to be read only, "1" indicates that the data is to be read and written, and "3" indicates that the asterisk is to be ignored as a return parameter.

#### 7.5 Identification of Nodes Requiring Multipoint Constraint Equations

The procedure previously employed by REFIN to determine whether a newly generated nodal point requires a constraint or alternately, whether

the constraint on a node can be removed was described in detail in Appendix E of Reference 1. The only check to be performed by subroutine MPC had been to determine if any independent nodes of a constraint equation were themselves dependent nodes. If they were, then the constraint equation was assumed to remain. This could result in the retention of unnecessary constraints as will be demonstrated in an example problem below.

An unrefined model for a square plate composed of 18 VAST triangular plate bending elements (IEC=4) is considered. The central element is first refined to order 2 and then the resulting four elements are refined again to order 2. The final mesh is shown in Figure 7.1. The refinement of the first element results in new nodes 17, 18 and 19 which are all border nodes and have constraints generated for them. The refinement of the first of the new elements (connectivity of 6, 17, 18) results in the new border nodes 20, 21 and 22 with constraints generated for them. Similarly, the refinement of the remaining new elements produces the new border nodes 23 to 28. It is clear that although the constraint equations on nodes 22, 23 and 24 would be retained as a result of their independent nodes being themselves dependent, the constraint equations are in fact not necessary.

As explained in detail in Chapter 4, this algorithmic deficiency has now been corrected. The REFIN program was modified to identify and remove the constraint on these nodes by using the concept of an "interface" node. An interface node has been defined to be a node whose constraint equation is common to two or more parent elements. The additional qualification on the check for constraint equation retention is that if dependent nodes are found in the constraint equation and the node is not an interface node, the constraint should remain.

Returning to our example problem, it can be readily demonstrated that the checks for constraint equation retention is now reliable. As

before, the refinement of the centre element produces new nodes 17, 18 and 19 which as border nodes have constraint equation generated for them. Again the refinement of the first of the new elements produces new border nodes 20, 21 and 22 with appropriate constraint equations generated. The refinement of the second new element results in new border nodes 23 and 24 having constraints generated. Node 22, however, is specified as an "interface" node since its independent basis is contained in more than one element. The refinement of the third and fourth new elements results in new border nodes 25, 26, 27 and 28 having constraints generated and 23 and 24 being flagged as "interface" nodes. (It is important to note that the logic for recursive refinement of finite element models is reliable only if no restarts are involved. Sufficient information is not stored for these operations to be carried out reliably if model refinement is restarted.)

The element refinement specifications are complete and now subroutine MPC is used to determine which constraints are required. Constrained nodes 17, 18 and 19 remain since the comparison against the connectivity list of the independent basis determine it to be found. Constrained nodes 20, 21, 25, 26, 27 and 28 remain since the independent basis of these nodes contain dependent nodes and these nodes are not interface nodes. The constraints on nodes 22, 23 and 24 were removed since the nodes are identified as interface nodes and have dependent nodes in their constraint equations.

#### 7.6 Graphical Boundary Condition Selection

When program REFIN is used to refine a finite element model, new nodes are inserted between the old ones and any regularity of node numbering is destroyed. Without extensive experience in the operation of the program, the user will not be able to readily establish node numbers on selected portions of the structure as required for instance when defining boundary conditions. Graphical aids of some sort are therefore

desirable, the minimum being an option to plot the refined model or a portion of it and to label the node numbers. A capability which is somewhat more sophisticated and automated has been developed within REFINe in a previous contract [1]. It utilizes the graphics cursor capabilities of the VASTG graphics program PLOTV1.

The graphical node point selection capability is optionally activated when defining boundary conditions after specifications for the refinement of a substructured or unstructured finite element model have been completed. Within the current contract, the subroutine BCOND in REFINe program has been extended to retain boundary conditions assigned to the original model and in the case of a substructured model to ensure that a node having boundary conditions applied becomes a master node (if it is not already a master node).

The operation of the program to define problem boundary conditions is described in detail in Appendix D of reference [1]. Basically, the REFINe program transfers control to the VASTG graphics program PLOTV1 to activate graphical node selection. This selection is performed by placing a "window" around the nodes of interest. However, if the boundary conditions are to be specified along a curved edge then this becomes a tedious procedure. A user may therefore define the boundary conditions using the VASTBC [2] program after the model has been refined. VASTBC provides the user with the option to select nodal points individually using the graphics cursor or multiply using the graphics cursor to define a window in much the same way that the capability in REFINe based on the program PLOTV1 does.

### 7.7 Treatment of Skewed Coordinate Systems

When boundary conditions are associated with spatial directions not aligned with global coordinate directions, a skewed coordinate system can be utilized to align model degrees of freedom with directions appropriate



for the definition of constraints. Such a capability has been implemented within VAST [3] and thus it is desirable that the effect of defining skewed coordinate systems be completely taken into effect when refining a model. The REFIN program previously had the capability to only transfer the skewed coordinate system data from the original model to the refined model. In this contract, REFIN was further enhanced to account for the effect of the skewed coordinate system in the constraint equation definitions.

#### 7.8 Improved Element Selection Capability

The REFIN program has been modified to provide the user the option to select elements graphically or through interactive prompting at each stage of the element refinement specifications. The program previously allowed the user only once to define the option for the selection of elements for refinement either by graphical means or through interactive prompting. After the option was defined, all subsequent element selections would have to utilize the same selection option.

The capability to select elements graphically is described in Appendix D of Reference 1. A file PREFIX.ELM with the elements sorted in ascending order with respect to first the superelement number, next the element group number, and finally the individual element number, is produced. The organization of data in the PREFIX.ELM file in ascending order proved extremely beneficial. In REFIN, it was possible to improve the program efficiency (to decrease CPU requirements) and to reduce the amount of I/O. This improvement is only realized in the cases where the window contains several element groups. The I/O reductions (and associated time savings) were accomplished by skipping two sections of the REFIN main program; firstly, the one that copies remaining element group data to the output file and secondly, the section that reverses the input and output file unit numbers before proceeding to refine another element group. The refinement proceeds sequentially through elements in the

input file as directed by the PREFX.ELM file until all elements are refined. The two steps which were skipped were generally necessary when selecting elements via prompting because each time the user was prompted for an element group to be refined, the reply could be an arbitrary group number and could therefore involve data at an arbitrary point on the USE file.

The procedure now employed to specify the elements through interactive prompting results in the generation of a PREFX.ELM file which is the same format as the file created when the elements are selected graphically. The refinement proceeds sequentially through the elements identified on the PREFX.ELM file and therefore the unnecessary data handling is eliminated.

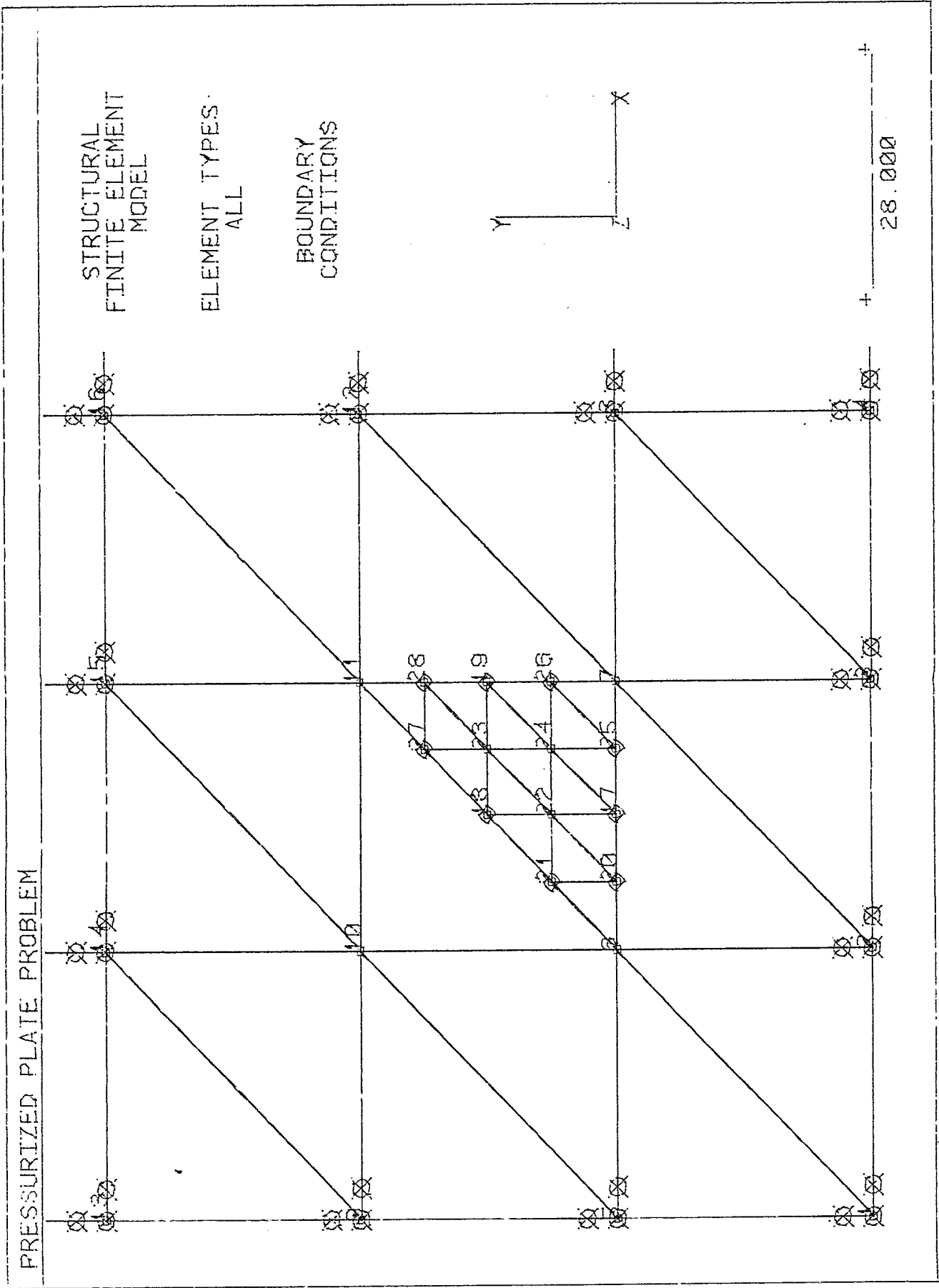


FIGURE 7.1: Recursively Refined Square Plate Model (◆ = Dependent Nodes)

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3. Vibration and Strength Analysis Program (VAST): User's Manual - Version #04. Martec Limited (January 1986).



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Program REFINE was initially developed as part of an adaptive finite element modelling package for VAST where from estimates of discretization errors based on element residuals, the finite element model would be selectively refined (by element subdivision) until acceptable levels of accuracy (according to computed error estimates) was attained [1]. However, increasingly the REFINE program finds application as a general modelling tool which complements the data generation capabilities of PVASt and HVASt. Under these circumstances, the PVASt and HVASt programs need only concentrate on achieving accurate geometric models. Since subsequent refinement of the model to attain acceptable finite element accuracy can be achieved with REFINE. This is particularly useful when the initial model is defined in a pointwise fashion with a graphics tablet. In this context as well, selective model refinement would generally be employed but the experience of the modeller would dictate where the model refinements would be.

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